Physics 15b Lab 4: Responses to Time Dependent Voltages

Chapter 8 in Purcell covers AC circuit. The chapter focuses on the response to sinusoidal signals $V = V_0 \cos(\omega t)$. If a circuit containing only resistors, capacitors, and inductors is attached to a voltage sources that produces a sinusoidal voltage as a function of time with a characteristic frequency $\omega$, the voltage and current for each circuit element must also be have the same frequency, though it may be phase shifted with respect to the original signal. Thus, it must be of the form $A \sin(\omega t + \phi)$, where $A$ is an amplitude that may be larger or smaller than the applied voltage $V_0$, and $\phi$ is the phase shift between the applied voltage signal and the current or the voltage across a circuit element. Voltages with $\pi/2$ phase leads and phase lags are shown below.

Providing only solution to sinusoidal input voltages may at first seem very limiting since there are not many truly sinusoidal signals in the world. In lab 2, you saw that even the voltage coming out of the wall in the science center is not completely sinusoidal. If there are no sinusoidal signals, why spend so much time on the response of circuits to signals they will never get? The answer is that any well behaved function of time can be expressed as a linear combination of Fourier components of the form $A \sin(\omega t) + B \cos(\omega t)$. If the function is periodic, then the linear combination is a sum over the Fourier series corresponding to the basic frequency of the function ($\omega$), as well as all of its higher harmonics ($n\omega$) where $n$ is an integer. The discussion below is copied from the Wikipedia.
Fourier's formula for $2\pi$-periodic functions using sines and cosines

The following formula, with arbitrary values of the sequences $\{a_n, n = 0, 1, 2, \ldots\}$ and $\{b_n, n = 1, 2, \ldots\}$ produces a periodic function, whose period is $2\pi$, on the domain $\{x \in \mathbb{R}\}$, the set of real numbers:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]. \quad (1)$$

If a function is square-integrable in the interval $[0, 2\pi]$, then it can be represented in that interval by the formula above, and the required sequences are given by:

$$a_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cos(nx) \, dx,$$

$$b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \sin(nx) \, dx.$$

Then formula (1) is called a Fourier series expansion of function $f(x)$, and the $a_n$ and $b_n$ sequences are called Fourier series coefficients.

Wikipedia uses the variable $x$ in the discussion above, but you can replace it with the time $t$ to get the expressions appropriate for the time dependent voltages being discussed here. The $n$th coefficient in the Fourier series correspond to the fraction of the signal that is at frequency $n$. The figure above shows a cosine wave as a function of time, its Fourier coefficients corresponding to the values $n$ as defined in the Wikipedia discussion above, and the spectrum of the cosine wave. The cosine wave consists of one single cosine, so the amplitude of the $n=1$ coefficient is 1, and the amplitude for all other $n$ is zero because the integral of the product of $\cos(w t) \cos(n w t)$ is 0 unless $n=1$. The spectrum, $g(w)$, is a function of frequency where the value of $g(w)$ corresponds to the amplitude of the corresponding Fourier coefficient. It again is a 1 for $n=1$, and a zero for all other $n$. This is consistent with the idea that the cosine wave contains only the single frequency $w$. A square wave as a function of time is the curve shown in the upper right. In contrast to the cosine wave, the cosine expansion for a square wave includes many frequencies, and all
of those frequencies contribute to the square wave signal. The amplitude of the
contribution of the higher frequency components decreases as $1/n$, so leaving out very
high n does not make a large difference to the final function. In the function generators
there is always a cutoff frequency, so the square wave that comes out of a signal
generator is only an approximation.

The principle of superposition implies that the effect of applying several different
voltages is simply the sum of the effects of each of the individual voltages. Thus,
the response to an arbitrary time dependent signal is the sum of the signals due to the
individual frequencies; therefore, there is no need to solve for every possible excitation,
one only needs to determine the response of the system to an periodic excitation, which is
done in chapter 8 of Purcell. Any arbitrary time dependent voltage can be expressed as
the sum over such individual terms. The solutions to each of these excitations is
determine in chapter 8. Given the individual current and voltage responses to each
frequency in the arbitrary signal, one can obtain the current and voltage response to the
arbitrary signal be adding up the known responses to each periodic voltage that appears in
the decomposition of the arbitrary signal. This is much easier than explicitly solving for
the current and voltage response of every time dependent voltage that is of interest.

Given the discussion above, it is sensible to consider the response of an electronic circuit
to a sinusoidal voltage at a particular frequency, and then to calculate the response to a
general time dependent voltage by expressing that voltage in terms of Fourier
components and using superposition to express the final result as the sum over the
response to each of the individual frequency components. A very nice discussion of AC
circuits excited by sinusoidal voltage is given at
http://www.physclips.unsw.edu.au/jw/AC.html

Pre-lab Questions:

1. Consider an RC series circuit. Let the drive voltage difference across the series
combination be $V_S=V_0\cos(\omega t)$. At what frequency is the RMS voltage across R
equal to the RMS voltage across C? For $\omega$ such that $1/(\omega C)>>R$, the RC series
combination can be replaced by a single circuit element. Is this element R or C?
Answer the next few questions by using that singe element replacement. What is
the phase shift, $\Delta\Phi$, between the voltage across this circuit element and the $V_S$?
What is the phase shift between the current, $I$, and the $V_S$? For $\omega$ such that
$1/(\omega C)<<R$, the RC series combination can be replaced by a single circuit element.
Is this element R or C? Fill in the table below by replacing the series circuit with
the appropriate single element, either R or C. (Hint: see the supplemental
information section on driven RL and RC circuits).

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$V_R/V_S$</th>
<th>$V_C/V_S$</th>
<th>$\Delta\Phi$ I and $V_S$</th>
<th>$\Delta\Phi$ I and $V_R$</th>
<th>$\Delta\Phi$ I and $V_C$</th>
<th>$\Delta\Phi$ $V_S$ &amp; $V_R$</th>
<th>$\Delta\Phi$ $V_S$ &amp; $V_C$</th>
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<td>$w&gt;&gt;1/(RC)$</td>
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2. Consider an RL series circuit. Let the drive voltage difference across the series combination be $V_S = V_0 \cos(\omega t)$. At what frequency is the RMS voltage across R equal to the RMS voltage across L? For $\omega$ such that $\omega L >> R$, the RL series combination can be replaced by a single circuit element. Is this element R or L?

Answer the next few questions by using that single element replacement. What is the phase shift, $\Delta \Phi$, between the voltage across this circuit element and the $V_S$? What is the phase shift between the current, I, and the $V_S$? For $\omega$ such that $\omega L << R$, the RL series combination can be replaced by a single circuit element. Is this element R or L? Fill in the table below by replacing the series circuit with the appropriate single element, either R or L. (Hint: see the supplemental information section on driven RL and RC circuits).

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$V_R/V_S$</th>
<th>$V_L/V_S$</th>
<th>$\Delta \Phi$ I and $V_S$</th>
<th>$\Delta \Phi$ I and $V_R$</th>
<th>$\Delta \Phi$ $V_S$ &amp; $V_R$</th>
<th>$\Delta \Phi$ $V_S$ &amp; $V_L$</th>
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<td>$\omega &lt;&lt; R/L$</td>
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<td>$\omega &gt;&gt; R/L$</td>
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3. What is the resonant frequency, $\omega_0$, for an RLC circuit with a 1000 Ohm resistor; a 68 mH inductor and a 1000 pf capacitor? What is the decay rate for this circuit? Bonus: At what frequency would the undriven circuit oscillate?

4. Consider the circuit described in question three driven by a voltage given by $V_S = V_0 \cos(\omega t)$. If $\omega = \omega_0$, so that the driving frequency is equal to the resonance frequency, what is the phase difference between the current and the voltage? (Hint see Figure 8.12 in Purcell). For $\omega << \omega_0$, what single circuit element (R, L, or C) is approximately equivalent to the RLC combination? What is the phase difference between the voltage and the current? (Hint see Figure 8.12 in Purcell). Is this answer consistent with the current to voltage phase difference for your single element replacement? For $\omega >> \omega_0$, what single circuit element (R, L, or C) is approximately equivalent to the RLC combination? What is the phase difference between the voltage and the current? (Hint see Figure 8.12 in Purcell). Is this answer consistent with the current to voltage phase difference for your single element replacement? Bonus: The phase difference between the voltage across the inductor and the voltage across the capacitor is independent of frequency. What is the phase difference between the voltage across the inductor and the voltage across the capacitor? On resonance, what is the ratio of the RMS voltage on the capacitor to the RMS voltage on the inductor? On resonance, what is the sum of the voltage across the inductor and the voltage across the capacitor? Let $V_R$ be the voltage across the resistor. On resonance, what is $V_R - V_S$?

5. Bonus: Consider a time dependent voltage given by $V(t) = \cos(\omega_0 t) \exp(-\gamma t)$, where $\omega_0$ and $\gamma$ are real numbers. Let $\omega_0 := 2 \pi$. Let $\gamma = 1$. Plot $V(t)$ when $t$ varies from 0 to 10. Let $\gamma = 0.1$. Plot $V(t)$ when $t$ varies from 0 to 10. The Fourier transform of $\cos(\omega_0 t) \exp(-\gamma t)$ is $\gamma / (\omega - \omega_0)^2 + \gamma^2]$. This function of $\omega$ is called a Lorentzian, and the linewidth (full width at half maximum) is $\gamma$. Given this information, sketch the frequency distribution corresponding to a time dependent voltage given by $V(t) = \cos(\omega_0 t) \exp(-\gamma t)$ for the cases where $\gamma = 1$ and $\gamma = 0.1$. 


Does the width of the frequency distribution increase or decrease with gamma? What is the Q of the two frequency distributions? (Hint: see equation 13 in Chapter 8 of Purcell) If \( \omega_0 = 100 \), sketch the frequency distribution corresponding to a time dependent voltage given by \( V(t) = \cos(\omega_0 t) \exp[-\gamma t] \) for the cases where \( \gamma = 1 \). What is the Q of the frequency distribution? What is the general relationship between \( \gamma \) and \( 1/Q \)? (Hint: see equation 47 of Chapter 8).

6. Bonus: Function generators produce time dependent voltages. They often offer a square wave, a triangle wave, and a sine wave. Please express a triangle wave as the weighted sum of sine waves. Plot the first harmonic alone. Plot the total signal that results if the first harmonic is removed from the triangle wave.
1. **RC Circuits with AC Voltage Inputs**
   a. **Materials:** function generator, oscilloscope, breadboard, micrograbber/BNC connectors, BNC cables, 0.1 microFarad capacitor, 1k Ohm resistor

   The initial setup for this experiment is very similar to the setup for the RC time constant experiment in Lab 2.

2. **Apparatus Assembly Directions**
   1. **Setup the scope to monitor the signal generator**
      a. Connect the BNC T to the output of the signal generator

   2. **Setup the signal generator to produce a 8 V amplitude sine wave at 1600 Hz**
      a. Make sure that none of the buttons highlighted by the lavender rectangles are depressed. If any are, press on the button and it should pop out.
      b. Adjust the settings on the signal generator to produce the appropriate signal.

3. **Connect the RC Circuit**
   a. Use the breadboard to connect a 1 k resistor in series with a 0.1 microfarad ceramic capacitor. Such a **ceramic capacitor** with axial leads is shown on the left above.
   b. Connect the output voltage of the signal generator in parallel with the series combination, as shown in the images above. Make sure the positive voltage (red micrograbber) is connected to the capacitor and the ground (black micrograbber) is connected to the resistor. Note: Channel 1 already displays the voltage across the RC series combination since it is the same as the output of the signal generator.

4. **Setup the scope to measure the voltage difference across and current flowing through the RC circuit**
a. Channel 1 already displays the total voltage difference across the series combination. Adjust the voltage sensitivity on channel 1 so that the amplitude of the signal is 3 boxes and the signal is centered on the screen.

b. Channel 2 will monitor the voltage across the resistor. Connect the micrograbbers from a micrograbber to BNC jack in parallel with the resistor, where the black micrograbber is connected to the same end as the black micrograbber from the signal generator. Use a BNC cable to connect this BNC jack to CH2 on the scope so that CH2 displays the voltage across the resistor.

c. Adjust the voltage sensitivity on CH2 so that the amplitude of the sine wave is at least 2 boxes. **What is the phase difference between the signal generator signal and the current in the circuit?**

d. Setup the math function to display the voltage across the capacitor.
   i. Press the red button highlighted by the red rectangle that is superimposed on the image of the scope that is shown on the left below.
   ii. Choose “-“ from the function menu so that the red display shows Ch1-Ch2 as shown in the image on the right below.
      1. If the menu comes up in FFT mode, press the bottom left menu button that is highlighted by the light green box, and the scope should return to the correct menu.
      2. Pick the correct source channels and arithmetic function in the Dual Wfm Math menu
         a. The first source should be set to Ch1 and the second source to channel 2, as shown in the image below.
         b. If either of the channels is wrong, press the menu button just to the right of the box showing the channel and keep pressing the button until your desired channel appears.

**Why is Ch1-Ch2 equal to the voltage across the capacitor?**
e. Adjust the voltage sensitivities on CH1, CH2, and MATH so that they are the same.

**Meaning of the scope display**
- Channel 1, which is shown in yellow, displays the voltage difference across the signal generator, $V_S$
- Channel 2, which is shown in light blue, displays the voltage across the resistor, $V_R$
- The math, which is shown in red, displays the voltage across the capacitor, $V_C$
- The current flowing through both $R$ and $C$ is the same and given by $V_R/R$.

**5. Setting up a the computer to capture an image on the scope screen (you don’t need to use this if you don’t want to, but it could prove a handy)**
- a. Open Safari web browser
- b. Type in scope IP address: 192.168.15.15
- c. Right-click image of scope screen and choose "Save Image to the Desktop".

**5. Begin data acquisition**
- a. Measure $V_R$, $V_C$, and the phase difference between the current and the voltage from the signal generator at 16 Hz, 1600 Hz, and 160,000 Hz. Change frequencies by pressing the frequency selection buttons indicated on the signal generator picture by the orange arrow and the two green arrows in the image at the beginning of this lab. The detailed instructions below are for the simplest data acquisition where the amplitude and phase are determined by counting boxes. The [supplemental information section on cursor use](#) provides an automated technique for taking this data. An even easier data taking option using 15b_RX_wfm2.vi is described just below this question. It works for both the RL and RC circuits.
  - i. Without changing the frequency knob, depress the 100 Hz frequency selection button to set the frequency of the signal generator to 16 Hz. **Fill in the row in the table below corresponding to 16 Hz.**
  - ii. Press the 10k frequency button, so that the frequency of $V_S$ is 1600 Hz. **Fill in the row in the table below corresponding to 1.6 kHz.**
  - iii. Press the 1M frequency button, so that the frequency of $V_S$ is 160 kHz. **Fill in the row in the table below corresponding to 1.6 kHz.**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$V_R/V_S$</th>
<th>$V_C/V_S$</th>
<th>$\Delta \Phi$ I and $V_S$</th>
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<th>$\Delta \Phi$ V_S &amp; $V_R$</th>
<th>$\Delta \Phi$ V_S &amp; $V_C$</th>
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<tbody>
<tr>
<td>16 Hz</td>
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<td>160 kHz</td>
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Are your results consistent with the RC series circuit as generalized voltage divider where each circuit element is modeled as a complex impedance? What is the measured complex impedance for the $R$ and $C$ in this experiment?

**Easy Data Taking Option** 15b_RX_wfm2.vi
You can get info at one frequency by calling the program after the signal is visible on the scope. If you keep the program open, you can get data at all frequencies by scanning the frequency with the program open. To take data, first find the frequency where the RMS voltage across R is equal to the RMS voltage across C. Turn the frequency down by a factor of 10, and begin scanning up in frequency by tuning the frequency control knob on the signal generator. Change scale when you run out of range on the knob. Continue until you reach a frequency that is 10 times the frequency where the RMS voltages were equal. You should get a nice plot, like that shown on the right above.

6. **Measure $V_C, V_R,$ and $I$ when $V_S$ is a triangle wave $V_S$.**
   a. Select the triangle wave output
      i. Set frequency generator to 160 Hz by pressing the 1 kHz frequency selection button
      ii. Set the time base so it displays ~ 3 periods
   Fill in the table below

<table>
<thead>
<tr>
<th>Frequency</th>
<th>VR Shape</th>
<th>VC Shape</th>
</tr>
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<tbody>
<tr>
<td>160 Hz</td>
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<tr>
<td>1600 Hz</td>
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<td>16 kHz</td>
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<td>160 kHz</td>
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</table>

Are your results consistent with the RC series circuit as generalized voltage divider where each circuit element is modeled as a complex impedance? Frequency filters are used to separate some frequency components of a signal from others. Can you explain your observations as the results of frequency filtering?

7. **Bonus: $V_C, V_R,$ and $I$ when $V_S$ is a square $V_S$.**
   a. Select the square wave output form the signal generator.
   b. Look at the signal at 16 Hz, 160 Hz, 1.6 kHz, 16 kHz, and 160 kHz by pressing the appropriate frequency selection buttons?

At what frequencies does the signal resemble the simple exponential decay characteristic of an RC circuit interacting with a DC voltage source? At what frequency does $V_r$ resemble a square wave? At what frequencies does $V_C$ resemble a square wave.

2. **RL Circuits with AC Voltage Inputs**
Materials: function generator, oscilloscope, breadboard, micrograbber/BNC connectors, BNC cables, 10 mH Inductor, 100 Ohm resistor

Apparatus Assembly Directions

1. Setup the signal generator to produce an 8 V amplitude sine wave at 1600 Hz
   a. Use the signal on the scope to adjust the signal generator to produce a sine wave with a 1 Volt amplitude and a 1600 Hz frequency.

5. Connect the RL Circuit
   a. Use the breadboard to connect a 100 Ω resistor in series with a 10 mH inductor. Such an inductor is shown in the image on the left
   b. Connect the output voltage of the signal generator in parallel with the series combination, as shown in the images above. Make sure the positive voltage (red micrograbber) is connected to the capacitor and the ground (black micrograbber) is connected to the resistor. Note: Channel 1 already displays the voltage across the RL series combination since it is the same as the output of the signal generator.

Meaning of the scope display
   a. Channel 1, which is shown in yellow, displays the voltage difference across the signal generator, $V_S$
   c. Channel 2, which is shown in light blue, displays the voltage across the resistor, $V_R$
   d. The math, which is shown in red, displays the voltage across the inductor, $V_L$
   e. The current flowing through both R and L is the same and given by $I = V_R/R$.

5. Begin data acquisition
   a. Measure $V_R$, $V_L$, and the phase difference between the current and the voltage from the signal generator at 16 Hz, 1600 Hz, and 160,000 Hz. Change frequencies by pressing the frequency selection buttons indicated on the signal generator.
picture by the orange arrow and the two green arrows in the image at the beginning of this lab. **Fill in the table below.**

<table>
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<tr>
<th>Frequency</th>
<th>$V_R/V_S$</th>
<th>$V_L/V_S$</th>
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<th>$\Delta \Phi$ I and $V_R$</th>
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<tbody>
<tr>
<td>16 Hz</td>
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<td>1.6 kHz</td>
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<td>160 kHz</td>
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</table>

Are your results consistent with the RL series circuit as generalized voltage divider where each circuit element is modeled as a complex impedance?

**3. Driven RLC**

a. Materials: 100 Ohm resistor; 1 k Ohm resistor; function generator; 68 mH inductor; 1000 pf capacitor. Scope.

**Apparatus Assembly Directions**

1. Setup the signal generator to produce a 1 V amplitude sine wave at 1600 Hz
   a. Use the signal on the scope to adjust the signal generator to produce a sine wave with a 1 Volt amplitude and a 1600 Hz frequency.
   b. Set the time base to 100 microseconds/box

2. Connect the RLC circuit as shown above
   a. Use the breadboard to connect the 1 kOhm resistor, BLAH capacitor, and 68 mH inductor in series
   b. Connect the micrograbbers from a micrograbber to BNC jack in parallel with the resistor
   c. Connect the output of the micrograbber/BNC jack output to the input of Ch2 on the scope using a BNC cable
   d. Connect the scope probe in parallel with the series combination of R and C as shown above.
      i. Connect the alligator connector from the scope probe to the grounded side of the resistor
      ii. Connect the micrograbber on the scope probe between the capacitor and the resistor

1. Setup the signal generator to produce an 8 V amplitude sine wave at 1600 Hz
   a. Use the signal on the scope to adjust the signal generator to produce a sine wave with a 1 Volt amplitude and a 1600 Hz frequency.
   b. Set the time base to 100 microseconds/box
2. Setup the RLC circuit shown in the schematic diagram above.
3. Setup the scope to measure the voltage difference across and current flowing through the RLC circuit
   a. Adjust the voltage sensitivities on Ch2 and Ch3 so that the amplitude of each signal occupies at least two boxes.
   b. Choose a timebase appropriate to the frequency
4. Call the Labview data acquisition program 15b_RCL.vi
   a. Use the mouse to adjust the mode control so that it is set to “acquire waveforms repeatedly”
   b. Keep the program open and it will plot the amplitudes and phase differences as a function of frequency.
5. Begin data acquisition
   Fill in the table below.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$V_R/V_S$</th>
<th>$V_L/V_S$</th>
<th>$V_C/V_S$</th>
<th>$\Delta \Phi$ I and $V_S$</th>
<th>$\Delta \Phi$ I and $V_R$</th>
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<th>$\Delta \Phi$ V_S &amp; $V_C$</th>
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</table>
Are there any voltages that exceed \( V_S \)? If so, explain how this could arise. What is the resonant frequency of the circuit? What is the corresponding gamma? What is the \( Q \) for the circuit?

6. **Sweep the frequency through resonance using the frequency control knob.**
   The Labview program will plot the information corresponding to the first row in the table above as a function of frequency, for frequency from \( \omega_0 - 2 \gamma \) to \( \omega_0 + 2 \gamma \) if you step through the frequencies in 2 kHz steps and press the acquire button. You do NOT need to write any of the information down, you can just display the plots that are produced by Labview. Note: 3 of the values are constant.

**Bonus:** Replace the 100 Ohm resistor with the 1 kOhm resistor and repeat.

**Bonus:** Change the signal generator output to a 1V square

**Bonus: Undriven RLC**

Materials: 100 Ohm, 1k,10,100k, 10 M Ohm resistors; function generator; 68 mH inductor; 1000 pf capacitor . Scope .

a. Set the signal generator to produce a 100 Hz square wave

1. **The RLC circuit is already setup, so no changes need to be made in the circuit**

2. **Setup the scope to acquire data**
   a. Adjust the voltage sensitivities on Ch2 and Ch3 so that the amplitude of each signal occupies at least two boxes.
   b. Adjust the timebase on the scope to100 microseconds/box
      i. You should see an oscillating signal an amplitude decay

2. **Call the Labview data acquisition program 15b_RCL.vi**
   a. Use the mouse to adjust the mode control so that it is set to “Fit decaying oscillation waveform”
   b. Keep the program open and it will plot the amplitudes and phase differences as a function of frequency.

What is the resonant frequency of the circuit measured by finding the time difference between two peaks on the decaying oscillation? What is the exponential decay rate? (Hint: find the time at which the amplitude is \( 1/e \) times the amplitude at \( t=0 \ ))What is the oscillation frequency according to the Fourier transform menu? What does theory predict the oscillation frequency to be? Explain any differences. .

b. The image below shows an annotated screen shot and a screen shot for the program, which provides all the information required to fill in the table.

<table>
<thead>
<tr>
<th>( V_R/V_S )</th>
<th>( V_L/V_S )</th>
<th>( V_C/V_S )</th>
<th>( \Delta \Phi )</th>
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</tbody>
</table>
4. Change the resistor to 10 kOhm resistor  
   a. Adjust the voltage sensitivities on Ch2 and Ch3 so that the amplitude of  
      each signal occupies at least to boxes.

5. Setup the scope to acquire data  
   a. Adjust the voltage sensitivities on Ch2 and Ch3 so that the amplitude of  
      each signal occupies at least two boxes.

6. Call the Labview program again

7. Try a 100 kOhm resistor and a 1 MOhm resistor

What is the resonant frequency of the circuits? What is the decay rate for the  
circuit? Fill in the table below and explain any differences from the result  
obtained using the 100 Ohm resistor.

<table>
<thead>
<tr>
<th></th>
<th>$V_S$</th>
<th>$V_R$</th>
<th>$V_L$</th>
<th>$V_C$</th>
<th>$\Delta \Phi$</th>
<th>$\Delta \Phi$</th>
<th>$\Delta \Phi$</th>
<th>$\Delta \Phi$</th>
<th>$\Delta \Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I and $V_S$</td>
<td>I and $V_R$</td>
<td>I and $V_L$</td>
<td>I and $V_C$</td>
<td>$V_S$ &amp; $V_R$</td>
</tr>
<tr>
<td>10k</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\Delta \Phi$</td>
<td>$\Delta \Phi$</td>
<td>$\Delta \Phi$</td>
<td>$\Delta \Phi$</td>
<td>$\Delta \Phi$</td>
</tr>
<tr>
<td>100k</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Adjust the signal generator so that there is a DC offset that places one of the  
square wave voltages at ground. Describe any changes from the case there the two  
square waves had non-zero values.

Double Bonus  
   c. Turn on math mode  
   d. Choose the Fourier Transform menu. A red Fourier  
      transform distribution should appear.
i. The image on the left below shows an example, where the red displays the Fourier transform.
ii. The horizontal is not well chosen, so it is difficult to distinguish the peak.

f. Adjust the horizontal scale so that you see a nice peak in the fourier spectrum.
g. Use the cursor in the Fourier Transform mode to find the peak in the spectrum. This should be equal to the oscillation frequency.

4. Ring Flinger/Gauss Gun
The ring flinger comes with many rings, some of which fly further than others. Try to optimize the distance flown. Note: liquid nitrogen is available to you for temperature control.
Challenge Problems:
1. Get biggest voltage EKG
2. Get EKG with the best signal to noise
3. Design a notch filter to remove 60 Hz.
4. Design a phase shifter that gives a 50 degree phase shift at 1 kHz.
5. Design a voltage multiplier that gives a gain of 10 from 10 Hz to 1 kHz
6. Design a voltage multiplier that gives a gain of 100 at 50 kHz.
7. Repeat challenge from the last lab to get the biggest signal across a wire loop with one disconnected end, but this time you man include any of the circuit elements from this lab.
8. Electro-optic modulator. An E/O is a device whose index of refraction changes with voltage. If you send light through it, you can rotate the polarization of the light. The problem with E/Os is that they require about 100 V to turn on and off, and most signal generators give only about 10 volts. The E/O is normally modeled as a capacitor. Design a circuit that will give a full modulation given a 10 V signal generator. Check your result using a photodiode.
b. Materials: metal loop; metal loop with a cut; AC Bfield generator (iron core transformer) Place the ring on the core and turn on the current. What happens? Most objects in the real world are not described by a single lumped circuit element. Even elements that are labeled as resistors, capacitors, or inductors really must be modeled using more than one element. For example, high value resistors are usually inductive, and almost all inductors are resistive. Given AC B field of the transformer, calculate the phase and voltage around the ring. Does the total magnetic field energy increase or decrease as the ring moves away from the surface of the transformer where the B field is largest. Does this explain what you observed?
Supplemental Information

Fourier Decomposition
The time dependent function \( f(t) \) is called the Fourier transform of the frequency composition \( g(w) \). The mathematics underlying Fourier transforms means that there is an uncertainty principle that applies even to classical signals: signals that are more spread out in time, have narrower frequency spectra; signals that are very narrow in time required many frequencies to add up and make the abrupt shape in time. In quantum mechanics, the time dependence and energy spread will have an uncertainty relationship because the energy spread can be expressed as a spread in frequency. Note \( \exp[-i w dt] \) is the time translation operator. Spatial Fourier decomposition is also used for spatially dependent function \( f(x) \) can be decomposed into its spatial frequency components \( g(k) \) where \( k \) is a wave vector there is a spatial uncertainty relationship that results in the diffraction of light in quantum mechanics this becomes an uncertainty relationship between position and momentum because momentum is proportional to \( k \). Note \( \exp[-i k x] \) is the space translation operator. The Fourier series can be expressed even more simply in complex notation. The Wikipedia discussion is included below.

We can use Euler's formula, \( e^{inx} = \cos(nx) + i \sin(nx) \), where \( i \) is the imaginary unit, to give a more concise formula:

\[
f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}.
\]

The Fourier coefficients are then given by:

\[
c_n = \frac{1}{2\pi} \int_{0}^{2\pi} f(x)e^{-inx} \, dx.
\]

The Fourier coefficients \( a_n, b_n, c_n \) are related via

\[a_n = c_n + c_{-n} \text{ for } n = 0, 1, 2, \ldots,\]

and

\[b_n = i(c_n - c_{-n}) \text{ for } n = 1, 2, \ldots\]
Recall that the response of an RC and LC circuit to a constant applied voltage was considered in Chapters 4 and 8 of Purcell. Below is a review of the voltage and current conventions for the three circuit elements, as well as the solutions for RC and LC circuits. In these cases, the initial condition of the circuit matters. For example, the result is different if there is an initial voltage across the capacitor than when there is no initial voltage across the capacitor.
RC and RL Undriven Solutions
(first order differential equations -> 1 free parameter for initial conditions, $V \to I-V/R$)

Without Battery
(energy stored in field is dissipated in heating the resistor)

Energy stored in Fields
$$\frac{1}{2} L I_o^2 \quad \frac{1}{2} Q^2/C$$

Energy is constantly lost, $I, V, E, B$ all approach zero

With Battery
(energy in battery is expended to create fields stored)

Energy is constantly lost, $I$ flows after $B$ becomes max

Energy is lost as $C$ charges, $I$ stops after $E$ becomes max

Characteristic time $\tau = L/R$

Characteristic time $\tau = RC$
Simplifying Using Equivalent Circuits

The best textbook on Electronics is almost certainly “The Art of Electronics” by Professor Paul Horowitz. He and Tom Hayes also offer a fantastic course in the physics department called Physics 123, that provides a thorough introduction to both analog and digital electronics. One tip that they offer is to simplify circuits by replacing combinations of circuit elements with simpler circuit elements that approximate the actual elements. An example for resistors is shown below. If two resistors are in parallel and one has a much larger resistance than the other, then the total resistance is approximately the same as the resistance of the resistor with the lower value. The reverse holds if the resistors are in series. This same logic can be extended to AC circuits if one replaces the circuit elements with their equivalent impedance.

1/R=1/R1+1/R2 \rightarrow R \approx R2

to first order R=R2(1-R2/R1)=0.9 R2 for R1=10R2 or 0.99 R2 for R1=100R2

Basis of voltmeter measurements

R=R1+R2 \approx R1

to first order R=R1(1+R2/R1)=1.1 R1 for R1=10R2 or 1.01 R1 for R1=100R2

Basis of ammeter measurements
Examples of equivalent circuits for RC and RL circuits for short times and long times is shown below.

**Limits of DC Driven RL and RC Circuits**  
(first order differential equations -> Only 1 variable voltage, 1 free parameter for initial conditions either V or I, the other is then fixed since, V -> I=V/R)

- **Without Battery**  
  (energy stored in field is dissipated in heating the resistor)

  - Small t
  - Large t

- **With Battery**  
  (energy stored from battery is transferred to fields)

  - Small t
  - Large t

A similar replacement of a circuit by a less complicated circuit can be done for the case where the applied voltage varies sinusoidally in time, as long as one uses the complex expressions for the impedance of the circuit elements. A summary of the impedance for resistors, capacitors and inductors is shown below.
An RC circuit is an R and a C in series. One can see from the figure above, that the impedance for the C approaches infinity in the limit where the frequency approaches zero. Thus, at low frequencies the series RC combination can be replaced by the capacitor alone. Similarly, at high frequencies the RC combination can be replaced by the resistor alone. The opposite obviously holds for inductive circuits. The results are shown below.
Summary of $V(t)$ & $I(t)$ in RL and RC Circuits in Limits $\omega \to 0$, $\omega \to \infty$

$I = V_0 / [R^2 + (\omega L)^2]^{1/2} \cos(\omega t + \phi)$

$\phi = -\arctan(L \omega / R)$

$V_L = \omega L V_0 / [R^2 + (\omega L)^2]^{1/2} \sin(\omega t + \phi)$

$\theta = \arctan(1 / (\omega C R))$

$V_c = -V_0 / [1 + (\omega RC)^2]^{1/2} \cos(\omega t + \theta)$

$I = \omega C V_0 / [1 + (\omega RC)^2]^{1/2} \cos(\omega t + \theta)$

$\phi$ is shift of $I$ from Drive. $V_R$ and $V_I$ always have a $\pi/2$ phase difference due to $\pi/2$ phase shift of time derivative.

High Frequency, $\omega$

$R/(L \omega) << 1$

$V_L \sim -V_0$

$I = 0$

$R/(\omega C) << 1$

$V_c \sim 0$

$V_0 = I R$

$I = 0$

$\lim_{\omega \to \infty}, \phi \to -\pi/2$

$I = 0; V_L = -V_0 \cos(\omega t)$

$\lim_{\omega \to \infty} \theta \to 0, V_c > 0$

$I = (V_0 / R) \cos(\omega t) = V_R / R$

Low Frequency, $\omega$

$R/(L \omega) >> 1$

$V_L \sim 0$

$V = I R$

$R/(\omega C) >> 1$

$V_c \sim -V_0$

$I = 0$

$\lim_{\omega \to 0}, \phi \to 0$

$I = V_0 / R \cos(\omega t) = V_R / R; V_L = 0$

$\lim_{\omega \to 0}, \theta \to +\pi/2$

$V_c = -V_0 \cos(\omega t); I = dQ/dt \to 0$
The voltages for underdamped and overdamped RLC circuits are shown below. For an underdamped circuit the algebraic form of the solution for the voltage across the capacitor is

\[ V_c = A e^{-\gamma t} \cos(\nu t + \phi) \quad \text{where} \quad \gamma = \frac{R}{2L} \quad \nu = (\omega_0^2 - \gamma^2)^{1/2} \quad \omega_0 = \frac{1}{\sqrt{LC}}^{1/2} \]

For the overdamped circuit the algebraic solution for the voltage across the capacitor is

\[ V_c = A_1 e^{-\beta_1 t} + A_2 e^{-\beta_2 t} \quad \beta_1 = (\gamma + \zeta) \quad \beta_2 = (\gamma - \zeta) \quad \zeta = (\gamma^2 - \omega_0^2)^{1/2} \]
Voltages vs Time
underdamped and overdamped from 0 to $2(2\pi)/\omega_o$

$\gamma=0.50$ $\omega=0.87$ $\phi=-0.17=-0.08\pi$

$\gamma=0.99$ $\omega=0.14$ $\phi=-0.45=-0.23\pi$

$\gamma=1.01$ $\zeta=0.14177$ $\beta_1=1.15$ $\beta_2=0.09$

$\gamma=2$ $\zeta=1.732$ $\beta_1=3.73$ $\beta_2=0.27$

$\gamma=5$ $\zeta=4.90$ $\beta_1=9.90$ $\beta_2=0.10$

$\gamma \to \infty$ $V_L$ and $VR$ have a quick time response, $VL$ decay time decreases, $VC$ and $VR$ decay time increases $\to$ at long times circuit $\to$ usual RC decay as if inductor were not present $VR$ also has a quick response at same rate as $VL$
Equivalent circuits for driven RLC circuits are shown below.

**RLC Limits**

**Low Frequency** $\omega << \omega_0 = 1/(LC)^{1/2}$

$I = 0$
$V_R = 0$
$V_c = -V_{in}$
$V_L = 0$

**High Frequency** $\omega >> \omega_0 = 1/(LC)^{1/2}$

$I = 0$
$V_R = 0$
$V_c = 0$
$V_L = -V_{in}$

**On Resonance** $\omega = \omega_0 = 1/(LC)^{1/2}$

$I = V_{in}/R$
$V_R = V_{in}$
$V_c = -V_L$
$V_L = V_o \omega_o/(2\gamma)$
$\gamma = R/(2L)$

Voltages across C and L are equal and opposite and +/- 90 degrees with respect to $V_o$. They can also be much larger since

$V_c = -V_L = V_o \omega_o/(2\gamma)$  where $\gamma = R/(2L)$

**HUGE VOLTAGE GAIN IS POSSIBLE!**
Voltage Gain of 50 on C and L, No Voltage gain on R
Cursor Use Instructions

Setup the vertical cursors on the scope to measure the phase difference between two sinusoidal voltages at the same frequency

a. Press the cursor button on the scope, which is highlighted by the light blue rectangle in the image above. This should bring up the cursor menu shown in the screen shot on the left below.

b. Use the menu buttons to the right of the display to choose the “V Bars” option. This menu item is highlighted by the bright green rectangle in the image on the left above.

d. Press the menu button below “Function V Bars”. This button is highlighted by the orange rectangle in the image below. Pressing this button will bring up the menu selection for the vertical bars. This will bring up the menu shown in the image that is on the right above.

e. Choose the “Phase” option that is highlighted by the pink rectangle

d. Tell the scope what the period of your signal is

   a. Use the “Select” button highlighted in dark green to toggle between the cursors
   
      ii. Use the knob highlighted in purple to position the cursors.
   
      iii. The upper line in the cursor display which begins with Δ, the greek capital Delta gives you the phase difference between the two cursors.
   
      iv. Move one cursor to a waveform maximum
   
   e. Press the select button that is highlighted on the oscilloscope. This selects the other cursor

   f. Move the second cursor the one of the neighboring waveform maxima

   g. Press the menu button next to the “Use cursor Positions as 360 degrees” option that is highlighted by the bright green rectangle e

   i.
5. Setup the horizontal cursors on the scope to measure the ratio of the amplitudes of two sinusoidal waveforms (Skip this step if you just want to measure the amplitude manually, but it will give you faster data acquisition if you use the automated process)

a. Use the menu buttons to the right of the display to choose the horizontal bars option that is highlighted by the bright green rectangle in the image on the left above.

c. This will bring up the menu shown on the right side of the screenshot shown on the right above.

d. Choose the “Ratio” option that is highlighted by the pink rectangle.

e. Press the menu button below “Function H Bars” again. This should return you to the screen shown on the left above.

e. Position one cursor at V=0, so the voltage difference between this cursor and the other cursor is simply equal to the voltage of the second cursor.

i. Since the amplitude of a waveform is the voltage difference between V=0 and the maximum of the waveform, you can just leave this cursor at V=0 and never move it. The difference in position between this cursor and the other horizontal cursor will always be the amplitude of your waveform as long as the waveform is centered on V=0.

f. Bring the second cursor onto the display

i. Press the “Select” button on the scope. This button is highlighted by the dark green rectangle in the image on the left above.

1. The cursor that you had been using before should now display as a dotted line, indicating that it is no longer the selected cursor. The dotted line should be located at V=0

2. The newly selected cursor should appear as a solid line.

g. Make sure VR and VS are on the same voltage scale.

h. Set the voltage value that corresponds to 100%

i. Move the cursor to a peak of VS and press the menu button next to “Use Current Cursor Position as 100%”. This item is highlighted by the orange rectangle in the screenshot above.

i. Measure the ratio of any other voltage (e.g. VR) to the voltage value that you just chose as 100%

i. Translate the cursor until it is at the peak of VR. The ratio of the amplitude of VR to VS will be displayed in the cursor display. Should you
wish to get other ratios, such as VC/VS and VL/VS simply translate the
cursor to the peak values for those functions.

j. Turn the menu off, so that the cursor info displays in the menu area
i. Press the “menu off” button at the lower right hand side of the scope menu buttons.