Lab 1. Oscillations

In this lab you will look in detail at two of the most important physical systems in nature, the damped harmonic oscillator and the coupled oscillator. These systems appear over and over again in many different fields of physics: exciting atoms with a laser, crystal oscillators in computers, and playground swings. The lab will give you a chance to get your hands on physical models of oscillating systems, and get some practice making measurements on them.

There will be two sets of measurements and discussion. First, you will measure the time domain behavior for single and double oscillators (free decay, phase and amplitude for steady state driven solution, resonance, modes of two coupled oscillators, beating.) Then you will study the same systems in the frequency domain. This means learning how to interpret Fourier transform graphs generated by the data acquisition software. Finally, we will look briefly at the behavior of three and four coupled oscillators, introduce the idea of modes, and demonstrate how these ideas are related to modes of violin strings and bodies.

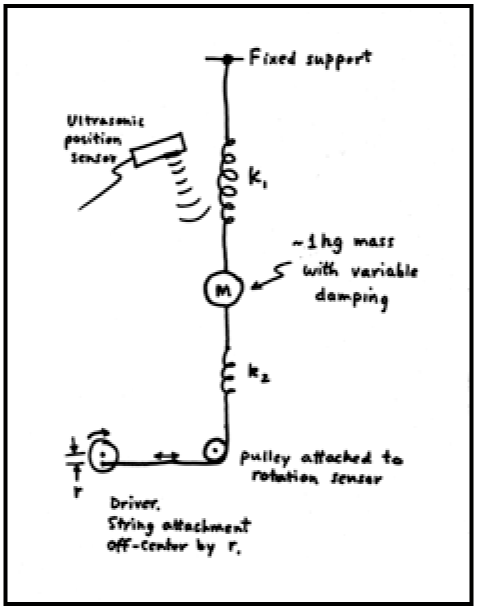
Learning Goals

In this lab session, students will:

- Make a physical, experimental, and visual (graphical) connection to theoretical concepts from lecture:
    - Normal modes
    - Resonance and damping
- Learn how to design an experimental protocol to characterize the behavior of an oscillatory system.

Part A: The driven damped simple harmonic oscillator

Apparatus



A steel mass is suspended from a fixed support with a spring (k_1). A driver is attached from below using a weaker spring (k_2). With this setup you have a very stable driving amplitude and frequency, the ability to observe large changes in oscillator amplitude, and a damping constant that can be controlled over a range greater than 1000.

The stepper motor slowly turns a disc (at a rate that you can control). It is controlled by a microstep driver configured for 1/64 microstepping. This means that when the controller sees 64 TTL pulses at its input, it turns the motor by one step, 1.8 degrees. From this you can calibrate the driving frequency in terms of the TTL frequency, a number determined precisely by a function generator.

The radius of the eccentric disc can be varied from zero to about 3 cm. A string attached to a pin in the disc rides over a pulley and couples to the mass through a light spring (k_2). This provides a sinusoidal drive with variable amplitude.

The damping method uses magnets attached to the mass to induce eddy currents in a nearby piece of aluminum. Those currents generate a force on the magnets that opposes the motion of the mass. Can you convince yourself, theoretically and/or experimentally, that this is a force proportional to the velocity of the oscillating mass?

Two transducers sense the position of the driver and the mass. The position sensor (Vernier motion detector 2) emits ultrasonic pings and infers distance from the time to receive an echo. A rotation motion sensor attached to the pulley gives an angle from which the driver displacement can be inferred. The diameter of the groove on the large pulley is 48 mm. Both sensors are interfaced to a computer running Logger Pro software.

Time-domain Challenge:

The challenge for the first part of this experiment is to predict and measure the oscillation frequency (ω_o), damping time (γ), and the quality factor (Q) for the damped-driven system.

Some things to try:

> ***Note: you don't have to do all of these!*** They are just different ways of exploring the system in the time domain.

- Play with springs, masses, driver and dampers to understand the system. Drive the oscillator by hand to feel the phase relationships between force and displacement at frequencies below and above resonance.
- Couple the eccentric driving pin to the mass with a light spring. Watch the pin and the mass and observe the phase relationships between force and displacement at frequencies below, above and near resonance.
- Use the loose magnets to experience forces due to eddy currents.

After you come up with a method to measure ω_o, γ, and Q, do the following:

- Change the damping to explore how it changes Q.
- Analyze the decay curve using the curve fit in Logger Pro.

Frequency-domain Challenge:

The goal here is to create a graph that characterizes the resonance linewidth of your damped-driven oscillator. To do this, you will design an experiment to collect data for a frequency vs. amplitude plot, like the one you examined in the Pre-lab. Compare your experimental data with the theoretical value for Q.

Some things to think about as you plan your experiment:

> ***Note: you don't have to do all of these!*** They are just different ways of exploring the system in the frequency domain.

- The simplest way to get the frequency vs. amplitude plot is to use a fixed-frequency sinusoidal driver, observe the steady-state solution, then vary the driver frequency by steps, and get phase and amplitude at each frequency. Can you think of other ways?
- A hint: What would happen if you excited the oscillator with a pulse. How can you get the amplitude as a function of frequency from this experiment?
- For lightly damped oscillator, record oscillations and watch the evolution of the Fourier Transform (FT) as the data is collected
  - What is the maximum frequency in the FT?
  - What is the spacing between frequency points?

- How does the width of the peak change as more data is collected?
- For stronger damping (still underdamped), record pulse force and response. Watch the evolution of the FT as data is taken.

Whichever method you choose to get the resonance curve, do the following to analyze it:

- Fit your amplitude data with the resonance function and extract γ and ω_0.
- What range of Q is accessible to this apparatus?

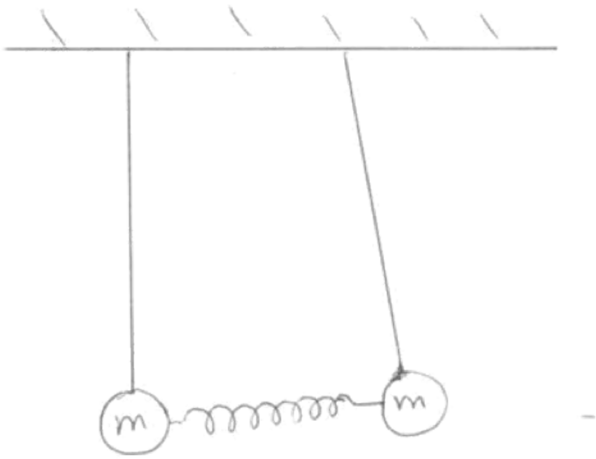
- Think about the sampling rate and number of samples to optimize the FFT in the region of interest.

References

- D. Kleppner and R. J. Kolenkow, An Introduction to Mechanics, (McGraw-Hill, New York, 1973) Chapter 10.
- J. B. Marion, Classical Dynamics of Particles and Systems, Second Ed., (Academic Press, New York, 1970) Section 3.5.
- K. R. Symon, Mechanics, Third Ed., (Addison-Wesley, Reading, 1971) Sections 2.9, 2.10.
- W. H. Press et. al., Numerical Recipes: The Art of Scientific Computing, (Cambridge Univ. Press, Cambridge, 1986) Section 12.2, Fast Fourier Transform.
- R. L. Burden and J. D. Faires, Numerical Analysis, Third Ed., (Prindle, Boston, 1985) Section 7.5 Trigonometric Polynomial Approximation (FFT).

Part B: Coupled pendulums

Apparatus



Double pendulums folded up and held together with a rubber band "double" as physical pendulums. Each physical pendulum is mounted on a Pasco Rotary Motion Sensor. Light (k ~ 3 N/m) springs couple the pendulums. The position of the spring along the length of the pendulum determines the strength of coupling.

A prototype apparatus with adjustable length can perform the "coupled pendulums" but not the "double pendulums" experiments.

Time-domain Challenge

The first challenge for the coupled pendulum system is to predict and measure the "carrier" and "envelope" frequencies.

Things to think about and try:

- To figure out what is meant by "carrier" and "envelope", excite the coupled system in different ways and record what you observe.
- What is the natural frequency of one of the pendulums and how would you measure it?
- Try coupling two pendulums with a spring. Excite the normal modes one at a time and measure their frequencies with the curve fit feature of Logger Pro.
- Now try to measure and extract $\omega_{carrier}$ and $\omega_{envelope}$.
- What happens if you couple three, four or N pendulums? Try three. Use intuition to predict the modes.

Frequency-domain Challenge

Now couple three of four oscillators and use the FFT analysis to measure the normal mode frequencies. Compare these to the eigenvectors/eigenvalues derived in class.

Things to try and think about:

- Couple four oscillators How many peaks does the FT have? What do the relative heights represent?
- Three Oscillators. Go back to N=3 and find modes. Compare FT.
- Try to excite only one or two of the coupled pendulums. How does this change the FT?
- What information is missing from the FT analysis in Logger Pro?

References

- H. Georgi, The Physics of Waves, (Prentice Hall, Englewood Cliffs, 1993), Chapter 3.
- F. Crawford, Waves: Berkeley Physics Course, Volume 3, (McGraw-Hill, New York, 1968) pp 32 - 36.
- J. B. Marion, Classical Dynamics of Particles and Systems, Second Ed., (Academic Press, New York, 1970) Chapter 13.
- G. Arfken, Mathematical Methods for Physicists, Fifth Ed., (Academic Press, San Diego, 1985) Chapter 14 \.
- W. H. Press et. al., Numerical Recipes: The Art of Scientific Computing, (Cambridge Univ. Press, Cambridge, 1986) Section 12.2, Fast Fourier Transform.
- R. L. Burden and J. D. Faires, Numerical Analysis, Third Ed., (Prindle, Boston, 1985) Section 7.5 Trigonometric Polynomial Approximation (FFT).

Part C: Infinite oscillators!

Check out the Falstad applets to investigate what happens as $N \rightarrow \infty$

- Fourier Series: http://www.falstad.com/fourier/
- Coupled Oscillators: http://www.falstad.com/coupled/
- Loaded String: http://www.falstad.com/loadedstring/

Note that you can study mechanical oscillators or musical instruments in more detail as your project at the end of the semester. Talk to Rob for more details.