

## Lab 3: Diffraction & Fourier Optics

This week in lab, we will continue our study of wave optics by looking at diffraction and Fourier optics. As you may recall from Lab 1, the Fourier transform gives us a way to go back and forth between time domain and frequency domain. Here we will explore how Fourier transforms are useful in optics.

### Learning Objectives:

In this lab, students will:

- Learn how to understand diffraction by extending the concept of interference (Huygens' principle)
- Learn how spatial Fourier transforms arise in optical systems:
  - Lenses
  - Diffraction gratings.
- Learn how to change an image by manipulating its Fourier transform.

### Huygen's Principle

Think back to the Pre-Lab assignment. You played around with the wave simulator (<http://falstad.com/ripple/>). Here you saw that we can create waves by arranging point sources in a certain way. It's done in technological applications like antenna arrays where you want to create a radio wave going in a certain direction. In ultrasound probes of the body an array of acoustic sources is used to create a wave that focuses at a certain point.

In the simulation we used sources with the same phase, but if you vary the phase, you can also move the focused point around. In 1678, Huygens figured out that you could calculate how a wave propagates by making a cut in space and measuring the phase and amplitude everywhere on that plane. Then you place point emitters on the plane with the same amplitude and phase as you measured. The radiation from these point sources adds up to exactly the same wave you had coming in (Figure 1). This is called the Huygens principle.

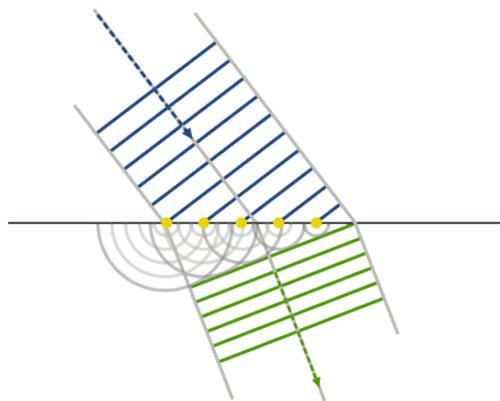


Figure 1: Wave refraction, according to Huygen's principle. (Image credit: [http://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel\\_principle](http://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle))

## Diffraction

As we saw on the simulations, when a wave hits an object on which the structure size approaches the wavelength of light, a diffraction pattern become noticeable. The Huygen's Principle is useful in understanding diffraction phenomena. Consider a plane wave incident on a single slit; all slits emit a spherical wave (see Figure 2). In other words, the interference pattern produced by a plane wave on a single slit is equivalent to the pattern produced by a point source.

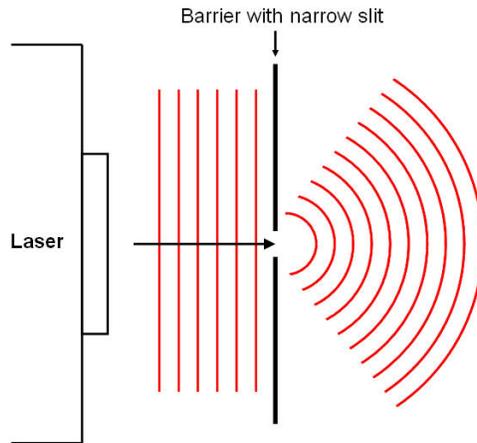


Figure 2: Single slit diffraction. The plane wave passing through the slit can be visualized as a wave emitted from a point source.

Consider the case of the double-slit experiment. The plane wave hits the slits and behaves like two point sources, which constructively and destructively interfere due to the difference in path length (Figure 3). A similar argument can be made for a diffraction grating, which is a series of slits (Figure 4). On the screen this interference pattern appears as a series of dots. The one that passes straight through is called the zeroth order.

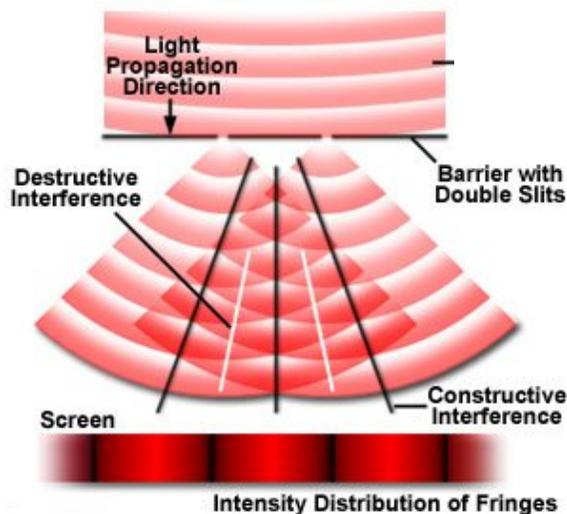


Figure 3: Double slit diffraction. The diffraction pattern is a result of constructive and destructive interference. (Image credit: <http://www.daviddarling.info/encyclopedia/L/light.html>)

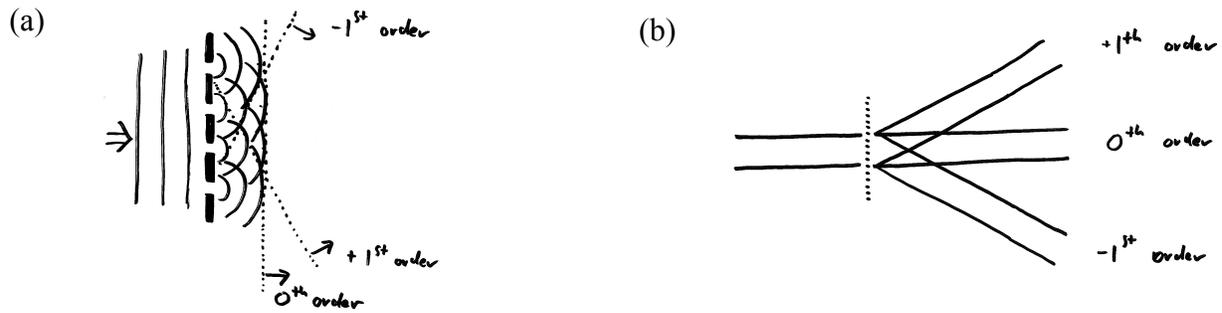


Figure 4: (a) In a diffraction grating, we can treat the slits as a series of point sources. The dotted lines denote wave fronts, i.e. positions of constant phase. (b) Series of diffraction orders ( $\pm 1^{\text{st}}$  order,  $0^{\text{th}}$  order, higher orders not shown).

## Part A: Diffraction Experiments

### 1. Fabric as a Diffraction Grating

Let's first have a look at a piece of fabric. The fabric acts as a "diffraction grating", a periodic array of holes through which light travels. The holes are the small open areas between the threads of the woven fabric. Each hole emits approximately a spherical wave, and all of these waves interfere on the screen and give rise of the interference pattern you observe.

1. Switch on the laser. **Keep the laser on during the entire lab, otherwise it might not restart for a while once it is warm, just like a video projector.** Close the aperture to create a beam with small diameter (approx. 1mm). Shine the laser onto a screen, about 2m away from the laser. Hold the piece of fabric into the laser beam, close to the laser source. What do you observe on the screen? Sketch the pattern. Discuss with your lab mates and TF what that pattern is, and briefly explain in your lab notes.
2. Hold the fabric in front of your eye and look at a bright LED light source at a distance. What do you see? Discuss with your lab mates and TF.
3. What happens if you use a fabric with a larger or smaller thread spacing? Use the various fabric samples to convince yourself that the pattern on the screen is not a shadow image; it is an interference pattern.

## 2. Measuring the wavelength of light

The interference pattern is due to the difference in path length that the light travels. For constructive interference, we want that path length difference (PLD) to be equal to an integral number of wavelengths ( $n\lambda$ ). If we assume the distance to the screen is large compared with the spacing of lines on the diffraction grating (a fair assumption in this case), then we can treat the outgoing rays as parallel and relate the path length difference to the wavelength ( $\lambda$ ) and spacing of the diffraction grating ( $d$ ), as shown in Figure 5.

$$PLD = n\lambda = d \sin \alpha$$

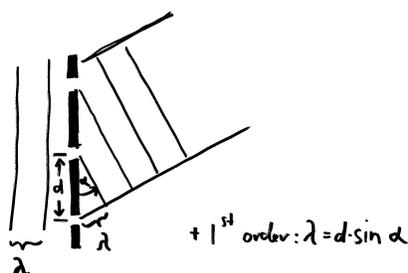


Figure 5: The diffraction orders appear at the angle ( $\alpha$ ) under which the path length difference of light coming from neighboring slits (spaced by  $d$ ) is a multiple of the wavelength of light ( $\lambda$ ).

We can now use a diffraction grating to measure the wavelength of our laser. The experimental set-up is shown below. Note that this angle ( $\alpha$ ) is equivalent to the one shown in Figure 5.

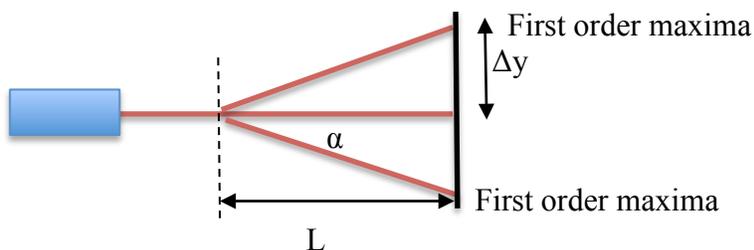
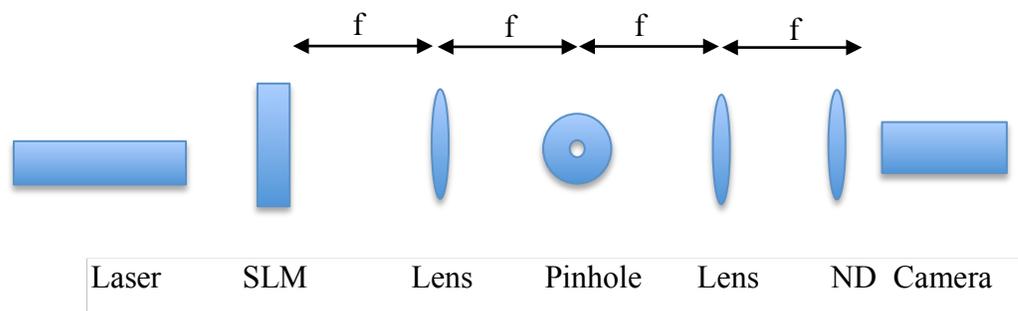


Figure 6: Experimental Setup for Diffraction Experiment

4. Use the diffraction grating in the slide frame and hold it into the laser beam. This mask has smaller slit spacings (**208 lines per mm**). **Make all required measurements to determine the wavelength of your laser light. Estimate the error by estimating how well you were able to measure the distances. Check with your TF.**

## Part B: Diffraction with the Spatial Light Modulator

In this apparatus, we have a laser that illuminates a Spatial Light Modulator (SLM) used in video projectors. It is a transparent object like a slide that displays the images on the computer screen. The optics following the SLM focus the image onto a screen. In this case, the “screen” is a camera (with the optics removed) that projects the image onto the second computer monitor. The neutral density filter in front of the camera can be used to control the intensity of the image. The schematic below shows the basic elements of this system.



*Figure 7: Fourier Optics System*

Three teams will do three different exercises. Be prepared to share your observations with the class. The number and width of slits can be controlled using the Matlab file on the computers.

1. The first group does single slit and varies the width. See how the distance between minima changes as width varies.
2. The second group does double slit and varies the spacing. See how the minima and maxima change as spacing varies.
3. The third group starts with double slit and varies the number of slits. See what changes as the number of slits vary.

## Fourier Transform

These diffractions patterns correspond to Fourier transforms. We saw in the first lab that the Fourier transform is useful for transforming a function of time to a function of frequency. One consequence of this is that periodic functions can be decomposed into a sum of sine waves. In general, the equations for a Fourier transform are given by:

$$f(t) = \frac{1}{2\pi} \int F(\omega) e^{-i\omega t} d\omega$$
$$F(\omega) = \int f(t) e^{i\omega t} dt$$

The single slit pattern is actually a Fourier transform. One way to think about this is to look at the transform of a square pulse. A square wave can be modeled by a sum of sine waves (Figure 8). A first order approximation uses a sine wave with the same frequency (the red line). The amplitude is a little larger than the square wave. You can see that the function is too short on the ends and too tall in the middle. To make the next approximation, add a sine wave with three times the frequency. This will increase the amplitude on the ends, and decrease in the middle (the yellow line). You can continue to correct the function in this manner.

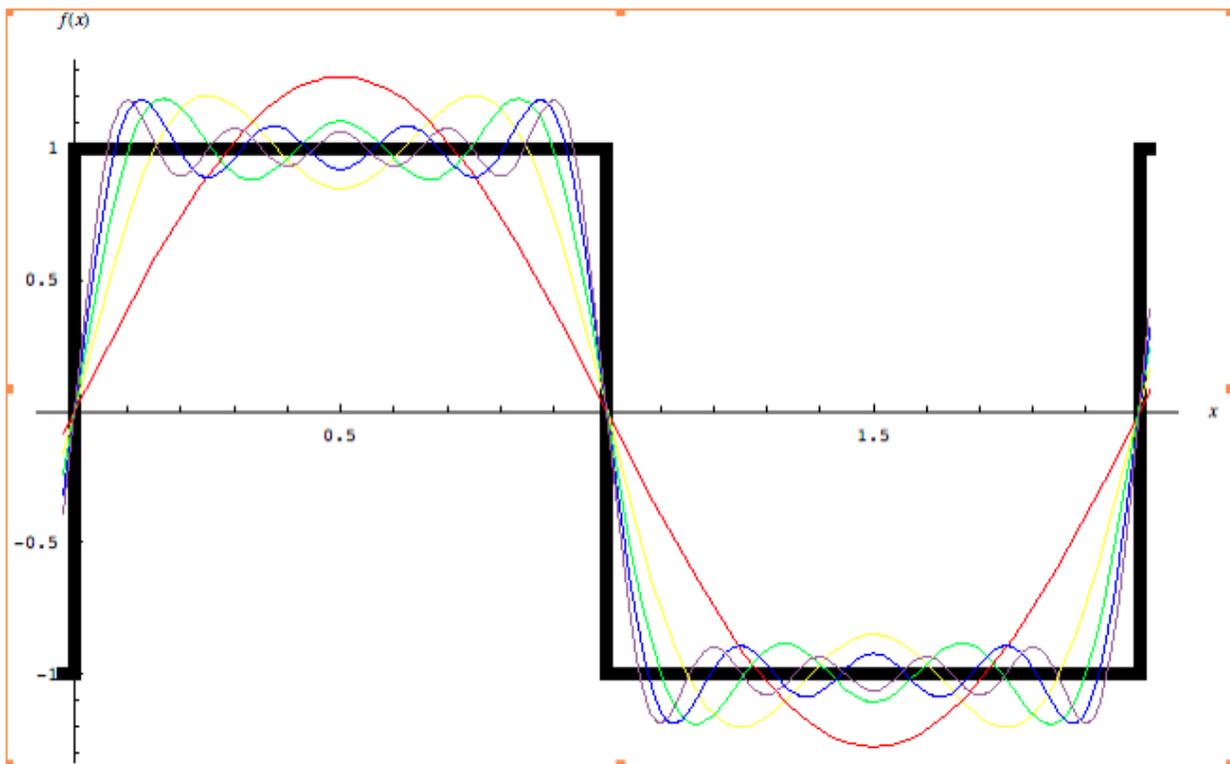


Figure 8: Fourier transform of a square wave. (Mathematica: FourierSeries.nb). <http://mathworld.wolfram.com/FourierSeries.html>

# Understanding Fourier Optics through Ray Optics

In this system, we have an image (from the SLM) at the focal length of a lens. One way to think about the action of the lens is that it turns information about the angle of a light ray into information about the position of a light ray. Conversely it also turns positions into angles (Figure 9).

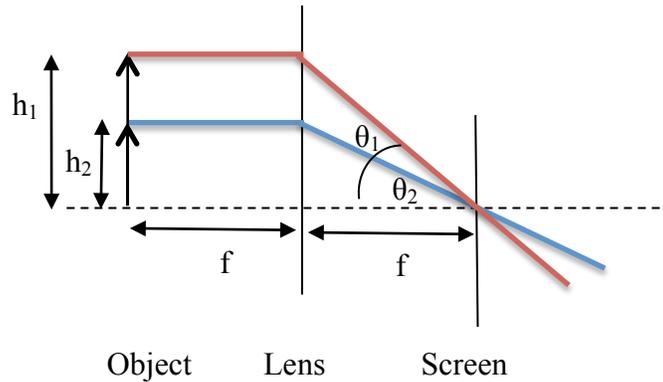


Figure 9: A lens produces a Fourier transform of an image. It converts spatial information (the height of the arrow) into angular information (the angle that the ray hits the screen).

Suppose we have two identical lenses (focal length  $f$ ) next to each other and the light comes from a certain position. The first lens sends this light to an angle. The second lens then changes the angle back to a position. So you get an image at a finite distance (Figure 10).

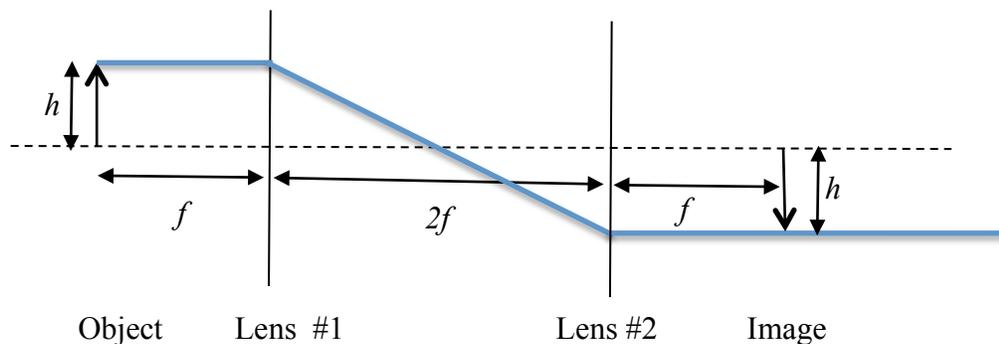


Figure 10: A two-lens system forms an image.

So what this essentially does is a **Fourier transform**. Recall our definition of Fourier transform. It makes a wavefunction of position into a wavefunction of angle. If the original image is a series of lines (as we saw above in Part B), it acts like a diffraction grating. The image formed by the first lens (at the aperture) is a diffraction pattern like we saw in Part A. We call this the *Fourier Plane*. Closing the aperture will block the higher order maximas, and thus give us an image that is fuzzy. This is equivalent to only using the first few terms in the Fourier transform of the square wave; we get something that sort of looks like a square wave, but without the crisp edges. Removing the central maxima will decrease the intensity of the final image. You will see the sharp edges, but without as much contrast.

In the setups we have in lab, the second lens is at a distance of  $2f$  behind the first lens. The Spatial Light Modulator (SLM), which serves as the object is  $1f$  in front of the first lens and the CCD camera is located  $1f$  behind the second lens. The camera has no lens so it acts like a screen; whatever picture we place on the SLM is imaged on the CCD.

To look at the Fourier plane, a third lens ( $f = 75$  mm) on a flip mount is located 75 mm from the CCD camera. The second lens ( $f = 200$  mm) converts positions *in the Fourier plane* to angles, and the 75 mm lens then converts the angles to positions on the CCD. So we can use it to look at the Fourier transform of the image on the SLM. A neutral density filter is the last element before the camera. The Fourier plane is much brighter than the image, so a polarizer on the same mount is used as a variable attenuator.

## Part C: Fourier Optics

Start by looking at a grating or a square wave (black and white bars) and identify the Fourier components that we saw in Figure 8 above. The grating is generated by Matlab code with variable number, width and spacing. Start with 15 slits, width of 20 pixels, spacing of 40 pixels.

- Manipulate the image by blocking certain components in the Fourier plane using the aperture and the pin. Try to look at individual Fourier components.
- Search for other pictures on the web and look at their Fourier transforms. Black and white, high contrast images work best.

## References

E. Hecht and A. Zajac, *Optics* (Addison-Wesley, Reading, 1975) Sections 11.1 - 11.3.2 and 14.1 - 14.1.3.