# Physics 15c Lab: The driven damped harmonic oscillator.

**REV1, 3/8/2007**

*This is a two-week lab; writeup due at the end of the second week, Friday, March 16, 2007*

In this lab you will look in detail at one of the most important physical systems in nature, the damped harmonic oscillator. This type of system appears over and over again in many different fields of physics: exciting atoms with a laser, crystal oscillators in computers, or even a child on a swing. We will use a damped mass-spring system to, hopefully, help give you both a quantitative and qualitative understanding. Please carefully review the section on the forced damped harmonic oscillator in Prof. Georgi's text book. Also review the attached handout: this is essentially the same material, but from a different textbook, just for variety.

In outline, you will work with one damping factor at a time, starting with the most heavily damped system. First, a free-induction decay (FID) will be produced and from this you will determine the Q (quality factor) of the system and its natural frequency (i.e. resonance frequency). Second, you will drive the system as a forced harmonic oscillator (FHO) at several different frequencies, measuring the phase (Φ) and amplitude (A) of the system in steady state. By phase, we always mean the relative phase between the position of the moving mass and the position of the drive. You will plot both phase and amplitude versus frequency, seeing the resonance shape and comparing the Q measured with FHO to the Q measured from the FID. These same measurements will be repeated for two other damping factors.

Which set of straws offers the most damping? Why?

**Brief Summary of the Harmonic Oscillator equations:**

Damped harmonic oscillator equation of motion: $\ddot{x} + \gamma\dot{x} + \omega_0^2 = 0$

Solution for displacement: $x(t) = Ae^{-\left(\frac{\gamma}{2}\right)t}\cos(\omega t + \Phi)$, where $\omega = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$.

For light damping ($\gamma << \omega_0$) the quality factor is $Q \cong \frac{\omega_0}{\gamma}$.

Equation of motion when driven by external force, $F_0\cos(\omega t)$: $\ddot{x} + \gamma\dot{x} + \omega_0^2 = \frac{F_0}{m}\cos(\omega t)$

Steady-state solution: $x(t) = A(\omega)\cos(\omega t + \Phi)$

Where $A(\omega) = \frac{F_0}{m}\frac{1}{\left[\left(\omega_0^2 - \omega^2\right)^2 + (\omega\gamma)^2\right]^{\frac{1}{2}}}$ and $\Phi = \arctan\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right)$.

In the underdamped approximation ($\gamma << \omega_0$) it is found from the amplitude equation that the full width at half-maximum FWHM $\equiv \Delta\omega \cong \gamma$, which gives $Q \cong \frac{\omega_0}{\Delta\omega}$.

## Instructions

You will take data for three different damping factors (three different sized straw bundles). Start with the biggest straw bundle, go through the below procedure to the end then switch to the middle-sized bundle, then finally the smallest bundle

1. Play with the mass and spring. Driving the system by hand (i.e., grab the spring and shake it up and down); can you make any observations about amplitude or phase in the high-frequency and low-frequency limit? Can you find resonance?

2. Attach the mass and spring to the driver. Pull the mass down near the bottom of its extension and let it go. Have the computer log the data. After about 3 damping times, where the amplitude has decayed by about a factor of $e^3$, stop logging data. Fit the data to a decaying exponential. NOTE: a calculated curve from the fit parameters will appear on the graph as a line. The automatic fit MAY have trouble; you can use the manual fit -- see the instructions below for help with manual fitting. Extract the resonance frequency ($\omega_0$) and the quality factor (Q) from the fit, and predict the full width at half-maximum ($\Delta\omega$).

Answer the following questions:

a. About how accurate are these determinations?
b. What is the major source of error in determining these values?
c. How many oscillations take place over the time that the total energy in the system decreases by 1/e?

3. Drive the system using the motor (connected to a rotary to linear motion adapter).

a. Start logging data and tune the frequency by hand, starting at about $2\Delta\omega$ below $\omega_0$ and ending well above $\omega_0$. Store the data and describe in words what you see. Find the APPROXIMATE resonant frequency and note the dial reading.

b. Based on your measurements made in FID, choose at least 10 frequency points that will range over the resonance (centered at $\omega_0$ and ranging from about $2\Delta\omega$ below $\omega_0$ to about $2\Delta\omega$ above $\omega_0$). The goal is to get a good measurement of both the drive and mass amplitudes and phase difference between the two versus drive frequency, $\omega$. In order to do this, you need to measure both the drive position and mass position and then fit both curves to extract the raw amplitudes and phases from each data set.

c. After you start driving the system at a particular frequency, wait for it to reach steady state. You can see this approach to steady-state by logging data and watching on the computer screen. Once in steady state, take sufficient data to get a good fit of amplitude and phase for driver and oscillator. Enter your data in the appropriate columns in the data table on page three of the LoggerPro file. The data table will automatically use the raw phase values for drive and mass and calculate the phase difference, $\Phi$, for you. Make a plot of amplitude A and $\Phi$, versus $\omega$. Make sure you took enough measurements near resonance to resolve any interesting features – ask your TF if you aren't sure about this. If you did not make enough measurements, go back and take some more.

4. After you have good plots of amplitude and phase for the three different damping factors, answer these questions.

a. At what frequency does the mass move when you are driving it?

b. What happens to the phase as you go across resonance? At what phase shift is the efficiency of putting energy into the system at a maximum? At what frequency?

c. From your plot, extract $\omega_0$, Q, $\Delta\omega$. Compare with the FID values.

d. Compare driver amplitude $A_0$ to the amplitude A both on and off resonance using the fit from both the driver and oscillator displacement. Compare $A/A_0$ at resonance to the Q.

## Instructions for data collection:

1. Determining the damped frequency and the damping time constant:
   a. Open the Oscillator_lab.cmbl file. The file is set up to take data in an appropriate way for the two sensors. If asked in a dialog box whether to use the detected sensors, click "OK". Rename the file ("save as...") to reflect your choice of damper.

   b. On "page 1" a graph is set up to show the mass position as measured by the ultrasonic detector. Set the mass in motion and press the "Collect" button. The following are some ways that you can change the way data is collected and viewed:

      - Change the number of points per second and the sample duration under "Experiment > Data Collection".
      - Use the Autoscale button: 
      - Use the mouse cursor to manipulate the axis scale by clicking near the arrows to offset or by dragging near the axis itself to change the scale factor.

   c. Collect a set of data for fitting to the decaying exponential function:

      - Save the data by "Experiment > Store Latest Run" or ⌘L. Open the data browser ("Data > Data Browser") and see that your data is now in a data set named "Run 1". Rename it to something more specific. Note that there is a new blank data set named "Latest" that is ready to hold the next set of data that you collect.

- Choose a region of the data to fit by pressing and dragging the cursor through the desired time range on the graph.
- Press the fit button: 

- Choose the fit function "Decaying Sine".
- Based on the data, estimate approximate values for the fit parameters. Then use trial and error in the manual fit mode to determine the "best fit" parameters.

2. Turn to "page 2" to collect data for the forced driven oscillator.
   a. Turn on the motor and collect data.
   b. Set data collection to take a long (1000 second) set of data. Collect data and watch the driver and oscillator displacements until you are convinced that transient behavior has been damped out.
   c. Stop data collection and start again(no need to save the old data), taking about 50 seconds of displacement data.
   d. Fit the data for driver displacement to a sine function, now using autofit. Repeat the same fitting procedure for the oscillator displacement.
   e. Record in the table on page three of the LoggerPro file the frequency, amplitude, and phase for the two signals.
   f. Save a representative data set. When you save a data set, you will have to reset the display options of your graph (by clicking in the graph area to get the "axes options" dialog box.) Saving takes several seconds, resetting the display takes time, so you may not want to save every data set. If you overwrite the display set by simply pressing "Collect" again, the new data is fitted and displayed automatically, since it is again called "Latest".
   g. Continue until you have all the data you need.

3. Use page three of the LoggerPro file to produce graphs of your data.

**General tips for Logger Pro**

1. If the auto fit does not work:
   a. Pass the cursor over a slightly different part of the graph while in the fit dialog.
   b. If that doesn't help, change to Manual fit, adjust parameters, change to Auto fit, press "Try Fit".

2. The data file structure can be viewed and edited by "Data > Data Browser".

3. To reset the choice of data set for display in a graph, double-click in the graph window to bring up the graph dialog box.