Physics 15c Lab 5: Measuring the Wavelength of Light with a Ruler

REV 0[1]; March 15, 2007

Due Friday, March 23, 2007.

The rest of the experiments in the course are take-home labs. There will be *help* lab sessions, scheduled for the first hour of some of the usual lab sessions (the help session can last longer than an hour, if you like, but *we'll expect you to show up in the first hour*; if you don't, we'll go home!) You may come to *any* help session.

These are the hours when we'll offer help:

- Wednesday, 7 pm
- Thursday, 4 pm, Thursday, 7 pm

As usual, we'll stay as long as you need help—but you must show up at the start of the session, please. You can come to *any* help lab session.

You will need to pick up a laser and a ruler, and a little bag of optics stuff. We hope you did that after lecture, Thursday, March 15. But if you missed that chance, email Rob Hart (hart@physics) to find out when you can pick up a kit. The laser is an expensive item[2], so we'll check you off as you return it along with the ruler, at term's end.

1 Introduction

Diffraction and interference describe effects unlike what geometrical optics would predict: the edges of shadows are not quite sharp; the beam passed by a slit is not a narrow rectangle. Diffraction patterns carry information about the spacing and location of the elements in the *diffraction grating* that produced them. Conversely, if we know the structure of the grating, we can deduce properties about the incident light, in particular its wavelength. This will be our task, in this first optics lab exercise.

The analysis of diffraction patterns is used extensively in the sciences to provide information about the microscopic structure of molecules, atoms, and nuclei. In addition to various forms of light (gamma rays, x-rays, visible light, infra-red, radio waves), even high-energy atomic and sub-atomic particles (electrons, protons, and neutrons) can be used in diffraction studies.

If one wants to know something about the wavelengths that make up a particular type of radiation (i.e., the *spectrum* of radiation), one could use an object such as simple diffraction parallel-slit grating in the form of a *spectrometer*. For example, molecules, atoms, and nuclei typically radiate or scatter radiation that corresponds to discrete frequencies and hence discrete wavelengths, λ. Knowing the spectrum (the intensity and wavelengths) of these radiations can tell us a lot about the molecule, atom, and nucleus under study.

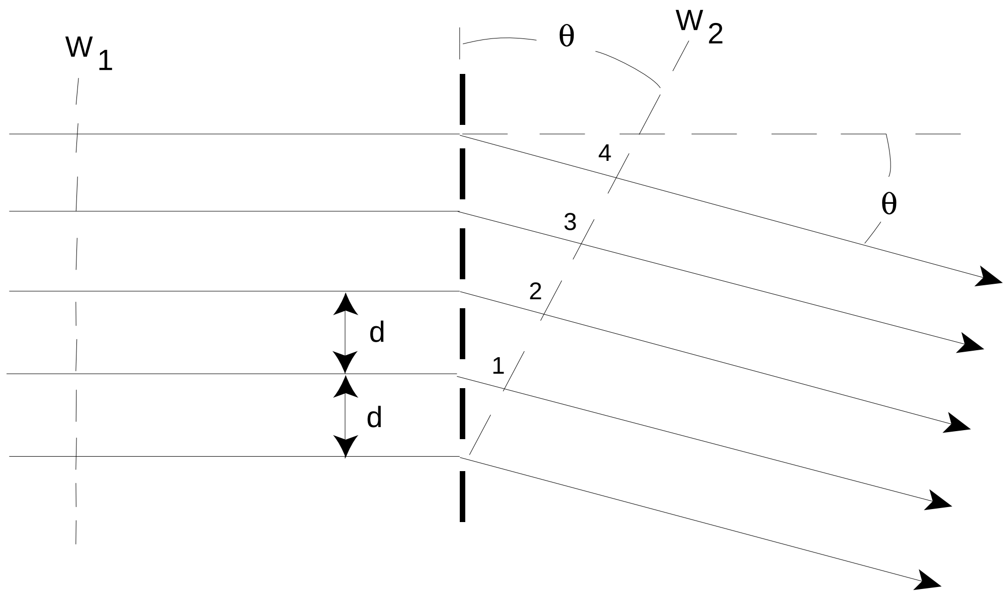
Since *diffraction-grating spectrometers* and other types of radiation spectrometers are widely used in *all* sciences you should be familiar with the basic physics of such devices.

[1]Revisions: this was numbered '6' in earlier terms (3/07).

[2]You may be inclined to protest that laser pointers are so cheap that every schoolchild has one. True, but the ones we provide are safer than the usual pointers: they emit lower power but are bright. They come to us from Australia!

2 The Diffraction Grating

We will consider first a transmission grating. The grating pictured below is like an almost-closed venetian blind with many long parallel slits perpendicular to the paper. The slits are uniformly spaced by a distance d. (We will not use the transmission grating in this experiment but because it is the easiest type of grating to explain we will describe it here as an aid to understanding the reflection grating. A transmission grating is supplied in your kit and will be used in a later experiment.)

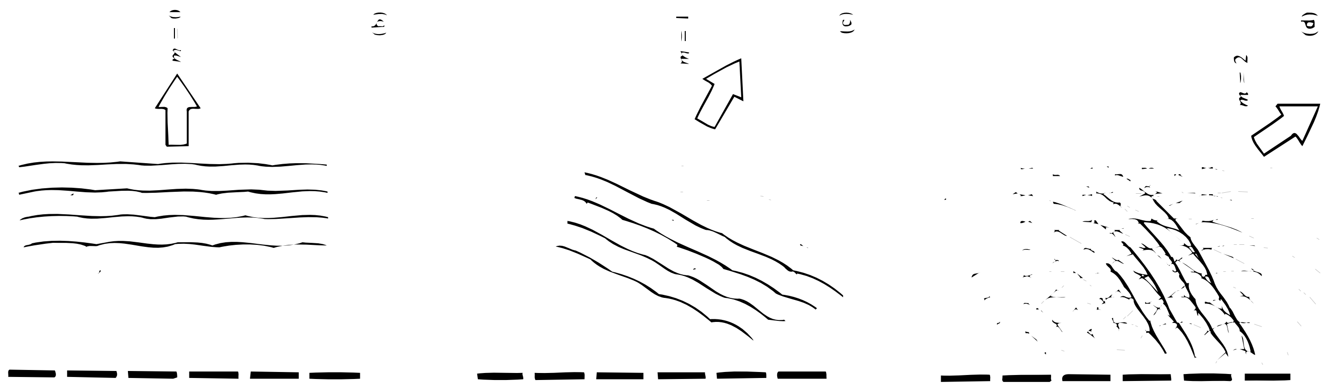


Consider now the ideal case of light of a single wavelength λ, travelling toward the right from a point source infinitely far to the left. We then have a wave front W_1, which is a plane $perpendicular$ to the direction of travel, and on which the phase is the same everywhere. The grating is placed parallel to W_1. On the right of the grating (according to Huygen's principle) wavelets of light spread out from each slit, and each of these wavelets retains the phase of the incident wave. The wavelets combine to make new wave fronts like W_2. For an angle θ such that successive ℓ's differ from one another by an integral number of wavelengths, the waves from each of the slits will be in phase and will interfere constructively. The smallest non-zero value of θ at which this can occur will be when $\ell_1 = \lambda$. Then from the figure, sin$\theta = \lambda/d$. It can also occur for $\ell_1 = 2\lambda$, 3λ,.... The general formula is:

$$\sin\theta_m = m\lambda/d \tag{1}$$

where m is an integer, referred to as the **order**. Thus parallel light (plane wave-fronts) of a given wavelength λ will be diffracted into directions, θ_m. If a light source emits several different wavelengths at the same time, there will be in each order (except the zeroth order) a set of values of θ_m corresponding to each wavelength. The set of wavelengths found in this ways is called the $spectrum$ of the particular source, and each individual component is called a *spectral line*. Each kind of atom has a characteristic spectrum. We find what elements are in stars by analyzing the spectrum of light for them. To obtain diffraction angles of few degrees, the spacing d must be $\sim 10\lambda$ or $\sim 5 \times 10^{-4}$ cm for visible light.

The figure below (from Hecht, Optics, p. 410) illustrates the effect again, but this time suggesting the way that the several point sources provided by the slits combine to produce plane waves in several directions–those directions where the paths from the several sources reinforce.



m = 0
(b)
m = 1
(c)
m = 2
(d)

3 Objective

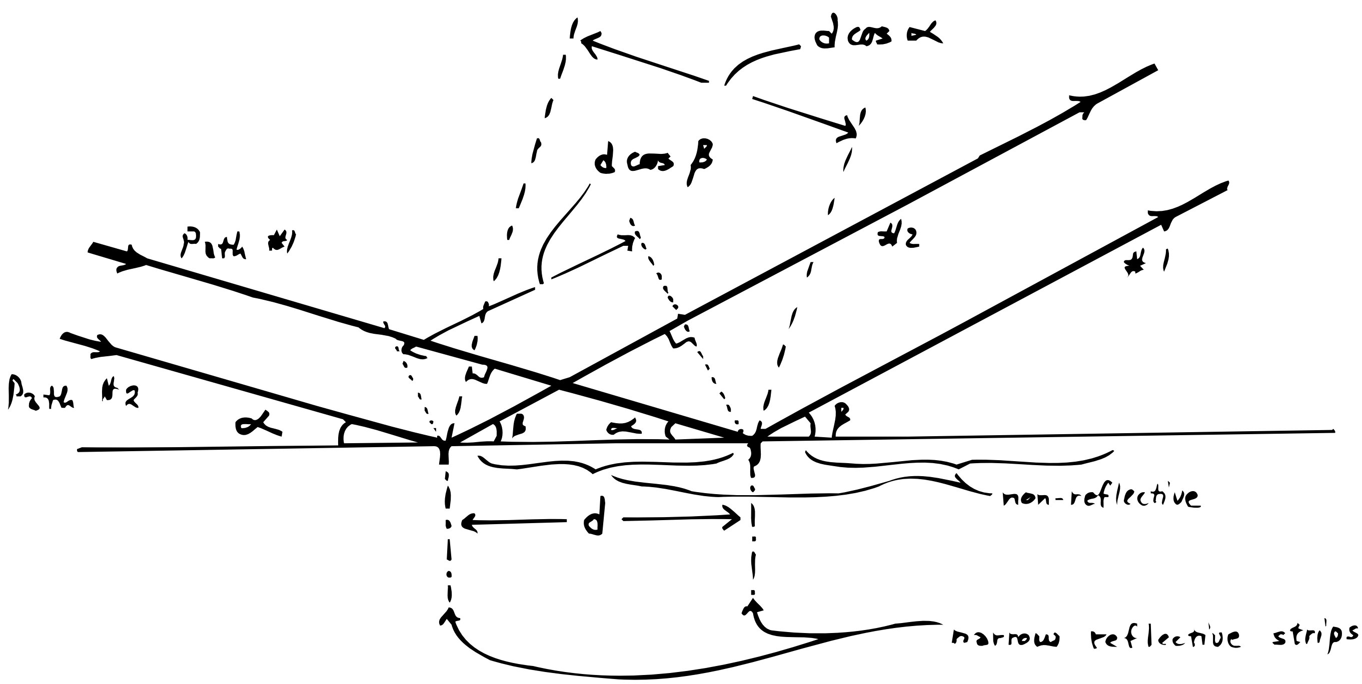
Measure the wavelength of light from a laser with an ordinary steel ruler[3]

4 Method

We will shine a laser at a low angle at a metal ruler, scored at 1/64" or 1/100" intervals. The experiment turns out to be a near-equivalent to passing light through a diffraction grating. Why this should be so is far from obvious. It turns out that again constructive interference can produce bright spots, and from these one can infer the light's wavelength. In the explanations below, we will sneak up on the actual experimental setup, approaching through another case that is easier to analyze.

4.1 An Explanation of a Similar Setup: Narrow Reflective Bands

Before we reach the actual case that you'll work with–shiny metal ruler scored with narrow non-reflective marks–let's assume an easier case: a non-reflective ruler marked with narrow shiny bands at the positions of the scorings on your ruler.



Again, the two incoming "paths" are parts of a plane wave (in phase, in your experiment, because they issue from a laser); the outgoing paths shown will be in phase as well–again forming a plane wave–if the difference between the

[3]The source of this exercise seems to be a 2-page article in American Journal of Physics by A.L. Schawlow, "Measuring the Wavelength of Light with a Ruler," 33 (11), 922-923 (1965)

two path lengths is an integral number of wavelengths. By measuring the angles, beta, that give this result, we will be able to infer the wavelength. That's the simple argument. The rest–not so simple–is just to spell out the geometry and some helpful approximations that make the calculation feasible.

A Preliminary Point: this is not reflection!

You notice, we're sure, that the angles alpha and beta are not equal: this is not reflection that we are trying to describe here. Instead, each shiny band on the ruler acts like a point source of light, giving a fresh start to a cylindrical wavefront; this wavefront combines with others to provide a plane wave in a particular direction, as suggested in the figure above that showed a transmission grating.

The Difference in Path Lengths

Path number 1 is longer *before* hitting the ruler, shorter *after* hitting it. As the diagram above indicates, it is *longer* by $d(cos\alpha)$, shorter by $d(cos\beta)$; the difference in path lengths, in other words, is $d(cos\alpha - cos\beta)$. When that difference is an integral number of wavelengths, we'll get a plane wave, formed by constructive interference. That wave will produce a bright spot (about as wide as the original laser beam) when projected on a wall.

So, the formula for the orders, a little more generally, is

$$m\lambda = d(cos\alpha - cos\beta) \tag{2}$$

When $\lambda \ll d$, as it is in this experiment, ($cos\alpha - cos\beta$) must be extremely small for this to be true if m is an integer ~ 1. However, due to the way in which cosine varies with the angle for small angles, α and β can be large enough for easy measurement, while ($cos\alpha - cos\beta$) remains very small. This is the secret of why the *grazing incidence* method will measure wavelengths very short compared to the grating spacing. It is best seen by using the approximation (first two terms in the series) for the cosine. For small angles, $\cos\alpha \equiv 1 - \alpha^2/2$. Substitution in Equation 2 gives

$$m\lambda = d((1 - \alpha^2/2) - (1 - \beta^2/2)) \tag{3}$$

Then we can approximate α and β by using the small angle approximation $\tan\theta = S/L \sim \theta$. This second step leads to

$$m\lambda = d(\beta^2 - \alpha^2)/2 \approx d(s_m^2 - s_0^2)/(2L^2) \tag{4}$$

where s_o, s_1,... s_m are the positions of the bright spots on the screen (see the figure below). Note that we use s_0 as a measure of α, the angle of incidence, not because we have any other interest in the angle of *r*eflection.

The small angle approximation tan$\theta = S/L \sim \theta$ works pretty well: if, for example, β is 3°(0.05 rad) and α is 2°(0.035 rad), both of which are large enough for convenient measurement, $(\beta^2 - \alpha^2)/2$ will be only 0.0006.

We could refer continually to α, the angle of incidence, and s_0, using the equation in the form shown as 4; thus we could make multiple calculations of λ, using several successive bright spots, each of which indicates a different integral number of wavelength differences. We could then estimate λ from these multiple readings.

There is an alternative, as well: we can, instead, use equation 4 but rewriting it so as to describe the difference in heights (displacements from ruler or table height) of *s*uccessive bright spots. In this form, the angle of incidence, α, subtracts out:

$$\lambda = d(s_{m+1}^2 - s_m^2)/(2L^2) \tag{5}$$

To see how this form is justified, one can imagine finding λ as a difference between the values calculated for 2λ and λ, using first and second orders–first and second bright spot and s_0. λ would be just the difference–and in calculating that difference, α would subtract out. This second form may be the more convenient one to work with. **Does it offer any advantage in error propagation, over the method that refers always to s_0?** (See note below on independence among measurements).

4.2 Amending the Explanation to Fit Our Setup: Scored Lines Do NOT Emit

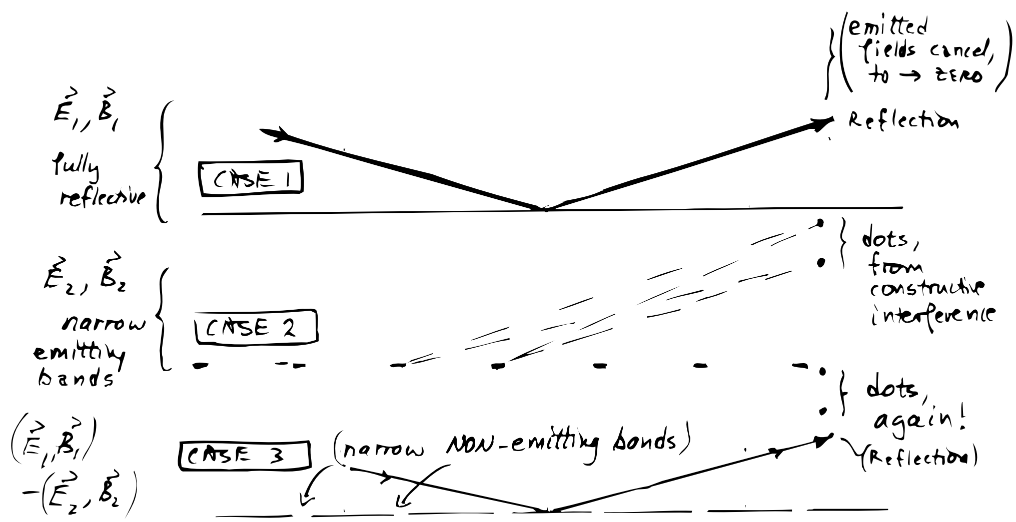
The explanation we just gave works nicely–for a different setup! Now we need to explain why it also works pretty well for what seems a complementary experiment: shiny ruler emitting at all but the narrow scorings. That, of course, is the setup we are actually using. How can the two arrangements be so nearly equivalent?

A Short Answer

If there were no scorings, one would see only the "specular reflection"– a bright dot coming off the shiny surface at an angle equal to the angle of incidence. That is a familiar result; using the terms of this discussion we would explain the absence of light elsewhere by saying that the many little emitters on the surface of the shiny ruler (like the sources of the wavelets in the standard explanation of the diffraction grating) interfere *destructively* everywhere except in that path. The dark scorings on the ruler eliminate some of the emitters, eliminating some of the destructive interference; hence the bright dots that now appear where a non-scored shiny sheet would have produced only darkness. [4]

A Slightly Longer Answer, Nevertheless Equivalent

Treating the scored ruler as non-scored ruler minus shiny bands [5]



[4]Thanks, for this elegantly simple explanation, to Wolfgang Rueckner's lucid notes for his Extension School course, Physics E1b.

[5]This is my attempt to restate the argument beautifully explained to me by Daniel Podolsky, Laboratory Teaching Fellow in this course (1998). Until Daniel corrected us, we had blandly treated the ruler as if the *scorings* were the only emitters: quite contrary to fact! It's true that the experiment allows us this pretense; but it's best to understand why.

Our experience with specular reflection reminds us that the emitted E and B fields above the *non-scored* shiny ruler are zero except in the reflective path. The fields above the set of narrow shiny bands show constructive interference that would produces a series of bright dots, on a screen placed to the right, as we have argued above. Superposition permits us to subtract one field from the other, so as to describe a third case, the case of our lab setup: this is, of course, the *shiny-and-scored* ruler.

What is called ”Case 3,” below, describes this scored ruler–and this Case 3 is just ”Case 1” (the non-scored surface) less ”Case 2” (the set of narrow emitting bands). Since Case 1 gives a field of zero, except at the angle of incidence, the difference field gives us that reflection plus a ”minus Case 2” field. So, we get the reflection, once more, and (since the sign of the ”minus Case 2” field does not matter–indicating only a phase reversal), again we get the projected bright dots of Case 2.

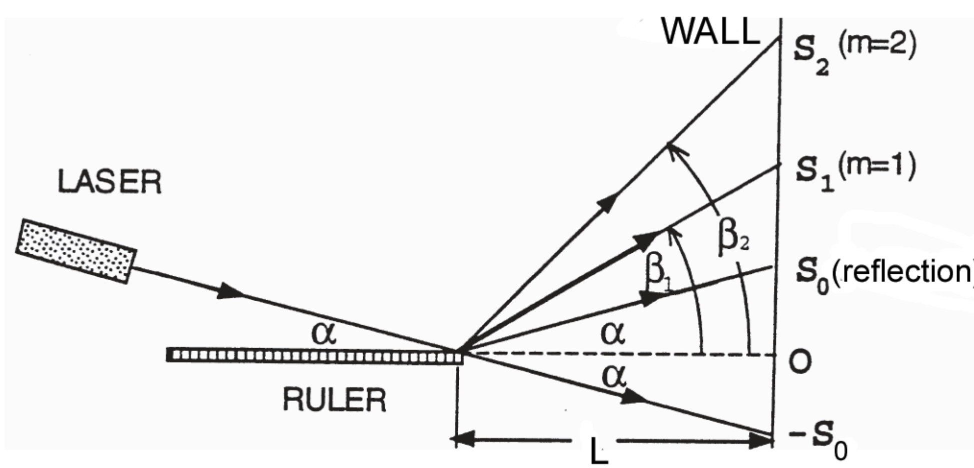
Still Another Way to Say This!

Another way to describe this subtraction of fields may be more direct: again treat the actual setup as the effect of the sum of two other fields. Again, one is the non-scored surface (Case 1); the other is whatever is necessary (Case Whatever) to produce the non-emission at scorings (Case 3), when summed with Case 1. What must *Case Whatever* look like? In order to get *no* radiation at the scorings, *Case Whatever* must show oscillators exactly out of phase with the emitters or oscillators on the non-scored surface.

So, ”*Case Whatever*” is the same as that of the narrow emitting bands– except that all the emitting oscillators are 180-degree shifted, or “flipped.” By itself, such a set of emitters would produce the same pattern of constructive reinforcement (bright dots) as that formed by the narrow shiny bands. Sum this output with the output of the non-scored, and you get those bright dots, plus the specular reflection.

Now, that explanation may be *less* clear than the one that spoke simply of subtracting fields. Take your pick.

4.3 Taking the Measurements



[6]The experimental arrangement is shown in the figure above. The ruler is placed on a table about 2 m from the wall and the laser is positioned so that the beam just strikes the end of the ruler at the grazing incidence. The laser can

[6]03/05 The light figure has been replaced. The previous figure was cut off at the bottom which created confusion.

be simply positioned by taping it down to a book. To vary the angle of the laser beam in a controlled manner, slide (something like) a stack of paper under the back of the book. The laser can be kept turned one using a clothes-pin. α should be $< 2°$, and the ruler must be perpendicular to the wall. (All angles in the diagram are shown greatly exaggerated.) Use a strip of paper on the wall for marking various spots. Record as many spots as you can. Notice that on your ruler there are a series of marks with different spacings and lengths interspersed. Verify which set of marks are producing the spots you see on the wall. (Slide the ruler left to right and see sets of spots appear and disappear.) You should use the most-tightly spaced rulings, which are the 1/64" marks (about 0.397 mm).

The dashed line is the extension of the ruler. *Do not* try to measure that point directly: instead, just find the midpoint between spots S_O and $-S_O$ (There is no light spot at O.) By letting the beam just touch the end of the ruler, $-S_O$, the direct beam spot is found. This is at the angle $-\alpha$ from the dotted line. At $+\alpha$ from the dotted line, S_O, the spot due to the mirror (specular) reflection from the steel is found. (This spot is not due to an interference phenomenon [7], and would be in the same place regardless of the wavelength.) Be sure to identify s_O correctly. To be sure, move the scale slightly sideways so that the shiny, unruled metal is in the beam. The diffracted rays, of order 1, 2, etc ; are found above S_O, as shown.

*C*aution: which ruler scorings are at work?

As you move the ruler sideways, if you go too far you may find unexpected bright spots appearing on the wall: halfway between the expected spots, and perhaps again halfway between those, though fainter. This happens if the laser beam wanders from the 1/64" rulings onto the wider spacings: the 1/32, and then 1/16" rulings extend farther across the ruler than the finer spacings, so that you can find yourself using these rather than the 1/64" rulings, by mistake. Make sure that you know *w*hich spacings you are using, as you measure the diffracted spots of the wall.

Negative orders?

What is *not shown* on the diagram above are orders of *negative m*; but is there any reason why we ought not to find path differences that are integral numbers of wavelengths *b*elow S_O? **Do you find bright spots indicating such orders?**

Use at least four *o*rders to form your estimate of λ. (Give the wavelength in units of nanometers.) Note sources of error and show how your final error bar for each calculation of λ was determined. Comment on your dominant source of error. Tape your strip of paper with your raw data into your notebook.

A Complication: Are your errors independent?

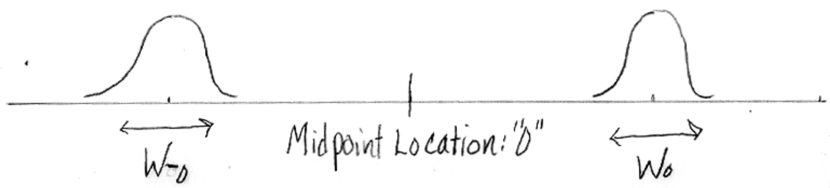
Watch out for a subtle problem: as you estimate the uncertainty of your λ measurement, probably you will be inclined to assume that your measurements are *independent*. To this point, we have relied on such an assumption, to allow us to sum the errors in quadrature, and then to average multiple readings. But can you do this here? Are your several calculations of λ independent, if based on several pairs of *o*rders? If they are to be independent, there must be no *common* measurements involved.

On the two sheets below are some tentative notes suggesting how you might estimate the uncertainty in measurement of a single *order*–and reminding us of the question of independence.

lb5_6_mar07c.tex; March 15, 2007: Notes on errors follow

[7] Except in the sense that specular reflection can quite properly be explained as a result of interference patterns, as noted above, sec. 4.2.1.

> Spot spread and Uncertainty for this Lab < $\frac{1}{2}$



→ Uncertainty of midpoint: $\delta_0 = \frac{1}{2}\sqrt{\left(\frac{W_{-0}}{2}\right)^2 + \left(\frac{W_0}{2}\right)^2}$

→ If we assume the same spread for each spot then we can define an uncertainty due to spread as $\delta_{spot} = \frac{W}{2} \Rightarrow \delta_0 = \frac{\sqrt{2}}{2}\delta_{spot}$

→ For any order (n) the uncertainty in the measure distance from the baseline to that spot is

$$\delta S_n = \sqrt{\delta_0^2 + \delta_n^2}$$

Uncertainty in baseline

uncertainty in the location of this particular spot.

We have already assumed that all spot uncertainties are equal therefore

$$\delta S_n = \sqrt{\left(\frac{\sqrt{2}}{2}\delta_{spot}\right)^2 + (\delta_{spot})^2}$$

$$\boxed{\delta S_n = \frac{\sqrt{6}}{2}\delta_{spot}}$$

All S_n uncertainties include a common uncertainty (δ_0) so they are not strictly independent.

2/2

The error in λ also includes contribution from the errors in L and d.

$$\lambda = \frac{d}{2L^2}\left(S_{m+1}^2 - S_m^2\right)$$

$$\delta\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial S_m}\right)^2\left(\delta S_m\right)^2 + \left(\frac{\partial\lambda}{\partial S_{m+1}}\right)^2\left(\delta S_{m+1}\right) + \left(\frac{\partial\lambda}{\partial L}\right)^2\left(\delta L\right)^2 + \left(\frac{\partial\lambda}{\partial d}\right)^2\left(\delta d\right)^2}$$

(I will calculate the first two terms. You need to calculate the L and d terms yourself)

$$\left(\frac{\partial\lambda}{\partial S_m}\right)^2\left(\delta S_m\right)^2 = \left(\frac{d}{2L^2}\right)^2\left(-2S_m\right)^2\left(\frac{\sqrt{6}}{2}\delta s_{pot}\right)^2 = \left(\frac{d}{2L^2}\right)^2\left(\sqrt{6}\, S_m\, \delta s_{pot}\right)^2$$

$$\left(\frac{\partial\lambda}{\partial S_{m+1}}\right)^2\left(\delta S_{m+1}\right)^2 = \left(\frac{d}{2L^2}\right)\left(2S_{m+1}\right)^2\left(\frac{\sqrt{6}}{2}\delta s_{pot}\right)^2 = \left(\frac{d}{2L^2}\right)^2\left(\sqrt{6}\, S_{m+1}\, \delta s_{pot}\right)^2$$

$$\left(\frac{\partial\lambda}{\partial S_m}\right)^2\left(\delta S_m\right)^2 + \left(\frac{\partial\lambda}{\partial S_{m+1}}\right)^2\left(\delta S_{m+1}\right)^2 = \left(\frac{\sqrt{6}\,d}{2L^2}\right)^2\left(S_{m+1}^2 + S_m^2\right)$$

$$\delta\lambda = \sqrt{6\left(\frac{d}{2L^2}\right)^2\left(S_{m+1}^2 + S_m^2\right) + \left(\frac{\partial\lambda}{\partial L}\right)^2\left(\delta L\right)^2 + \left(\frac{\partial\lambda}{\partial d}\right)^2\left(\delta d\right)^2}$$

Anita Bowles from notes written by Ehab Goldstein
03/05