A-1 Thermal and Shot Noise

From Physics 191r

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Author: Wolfgang Rueckner

First experiment: yes

Contents

Learning Goals

- Measure the charge of the electron and Boltzmann's constant.
- Become familiar with low-noise amplifiers and shielding techniques.
- Analyze frequency-domain data.
- Perform linear curve fits.
- Determine the temperature of liquid nitrogen by a thermal noise measurement.

Introduction

Intrinsic noise, random and uncorrelated fluctuations of signals, is a fundamental ingredient in any measuring process. This experiment is an investigation of two electrical noise phenomena: thermal noise and shot noise. Thermal noise is an energy equilibrium fluctuation phenomenon whereas shot noise involves current fluctuations which deliver power to the system in question. Both are inherent noise that is always present in a real electrical system and represent fundamental limitations and difficulties in making sensitive electrical measurements.

Thermal noise arises from the thermal fluctuations in the electron density within a conductor. In a formulation due to Nyquist (http://en.wikipedia.org/wiki/Harry_Nyquist) (1928), an idealized resistor is assumed to contain a voltage generator causing a fluctuating emf at the terminals, the mean squared value of which is given by the *Nyquist equation* [1]

$$\left\langle V^2 \right\rangle = 4 k_B T \Delta f\, R$$

where R is the resistance of the conductor, k_B is Boltzmann's constant, T is the absolute temperature, and Δf is the equivalent noise bandwidth[2] of the measuring instrument.

In the same year, Johnson (http://en.wikipedia.org/wiki/John_B._Johnson) experimentally verified[3] the dependence of the thermal noise voltage on the resistance (thus thermal noise is also known as *Johnson noise* or *Nyquist noise*). Thermal noise is independent of the material of the resistor and is constant with frequency ("white" noise) up to microwave frequencies; at higher frequencies the quantum energy hf of the oscillations becomes comparable with $k_B T$ requiring a modification of the Nyquist formula. Two derivations of the Nyquist formula are given in Appendix A and Appendix B.

Another random noise signal is due to the fact that, since the current is carried by individual electrons, there are rapid fluctuations about the average current. These fluctuations can readily by made visible in temperature-limited vacuum diodes, zener diodes, heated resistors, and gas discharge tubes. One of the most common is the temperature-limited vacuum diode, which will be referred to as a noise diode.[4] With no space-charge region around the cathode to smooth out the electron emission, the emission becomes a random statistical process. The instantaneous anode current deviates from the average value due to the discreteness of the electron's charge, and this fluctuation in the anode current is called **shot noise**. The mean squared value of the shot noise current is given by the *Schottky equation* derived by Schottky (http://en.wikipedia.org/wiki/Walter_H._Schottky) [5]:

$$\left\langle i_{sh}^2 \right\rangle = 2e\, i_{dc} \Delta f$$

where e is the charge of the electron, i_{dc} is the average dc diode current, and Δf is the equivalent noise bandwidth (ENB) of the measuring instrument. A derivation of the Schottky formula is given in Appendix C.

The amplitude distribution of the noise is Gaussian and independent of frequency from a few kHz to several hundred MHz. At low frequencies there is another noise, $1\,/\,f$ (flicker), that dominates.[6] In the high-frequency region (frequencies above that corresponding to the transit time of an electron across the gap of the diode), the output noise decreases because the flight of an electron is affected by the charge of other electrons likewise in transit. Experimental verification of Schottky's analysis was first provided by Hull and Williams. [7]

The noise sources mentioned above are incoherent and the total noise in a system is the square root of the sum of the squares of all the incoherent noise sources. In addition to these fundamental or intrinsic noise sources, there are a variety of other noise sources across the electromagnetic spectrum. These include noise from the power line frequency (and harmonics), AM, FM, & TV broadcasts, communication and computation devices, microwaves, etc, etc. Indeed, the amount of electromagnetic radiation surrounding you is phenomenal — electromagnetic interference (EMI) has become a major problem for circuit designers and its elimination (avoidance is a more appropriate word here) is crucial in this experiment. Ironically, even some of the instruments you'll be using to perform the measurements generate significant interference. EMI can be minimized with good laboratory practices. There are many ways in which these noise sources work their way into an experiment and some of the techniques used to reduce or eliminate them are described in Appendix 3 of the Lab Manual (https://coursewikis.fas.harvard.edu/phys191r/Cables_and_Shielding_Techniques) . Horowitz and Hill give a thorough discussion. [8]

In this experiment you will measure the thermal noise generated by several metal film resistors and the shot noise generated by a vacuum diode. By amplifying the noise voltage generated across a resistor with a low-noise preamplifier and spectrum analyzer, you can examine the thermal noise dependence on R and the shot noise dependence on i_{dc}.[9] In doing so, you will be able to determine the values of the fundamental constants k_b and e. Although the experiment is straightforward in concept, being able to measure Boltzmann's constant and the charge of the electron is not as easy as suggested by the Nyquist and Schottky equations. The experiment will give you a good introduction to the measuring process as well as dealing with noise sources. If you look at the papers by Hull & Williams (1925) and Stigmark[10] (1952), you will begin to appreciate the difficulty in achieving uncertainties less than 2% — happily, your job will be made a little easier in using state-of-the-art instrumentation. Nevertheless, you will learn firsthand that extraordinarily low-level signal measurements require extraordinary techniques as well as instrumentation.

Apparatus and instrumentation

The apparatus consists of three main units:

- thermal & shot noise generators (inside RF shield enclosure)
- low-noise preamplifier (Stanford Research 560)
- spectrum analyzer (Hewlett Packard 3561A)

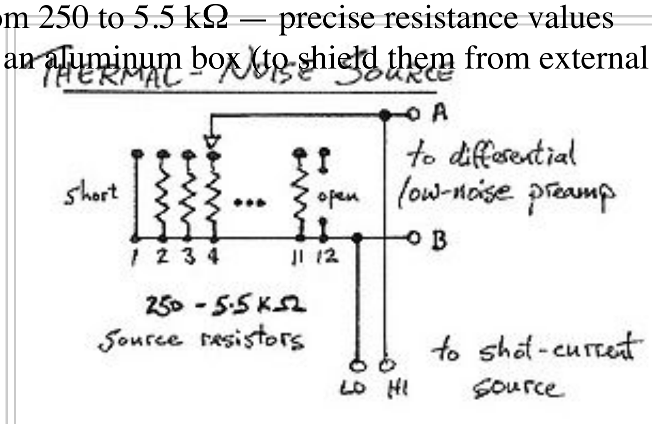
and ancillary instrumentation:

- noise diode filament power supply (Agilent E3610A)
- noise diode anode (plate) voltage supply (Keithley 2400 SourceMeter)
- RF signal generator (LogiMetrics 925-S125)
- digital multimeter (Keithley 169)
- oscilloscope (Tektronix 2215)

Noise generators

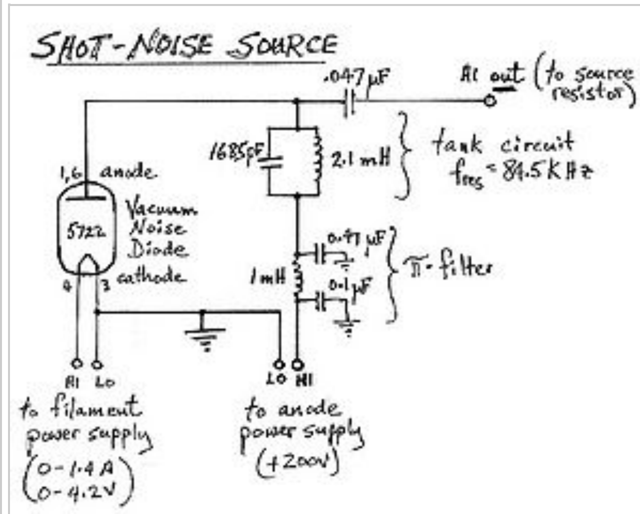
The circuits that produce the thermal and shot noise are diagrammed at right. A bank of resistors is used as the source of thermal noise and a type 5722 vacuum diode generates shot noise.

The noise resistors are standard metal film resistors, ranging from 250 to 5.5 kΩ — precise resistance values should be determined with a DMM. The resistors are housed in an aluminum box (to shield them from external RF noise) and selected by means of a twelve position rotary switch.



Thermal noise circuit

In order to make sure that the noise diode operates in the temperature-limited region, it is necessary to apply a fairly high dc voltage (+200 V) between the cathode (filament) and anode (plate). The dc current (i_{dc} in the Schottky formula) through the diode is measured by the SourceMeter, which also supplies the dc bias voltage. The shot noise current (ish) is made to pass through one of the thermal noise resistors. By measuring the voltage across this known resistor, one can determine the noise current through it. The resistor has to be large enough for an appreciable noise voltage to develop across it. On the other hand, if it's too large, the (dc) voltage drop across it becomes considerable and makes it difficult to maintain the anode at a sufficiently high voltage. To get around this difficulty, a tuned circuit is placed in the anode circuit of the diode, and the noise resistor is connected in parallel with it. The tuned circuit has a very low dc resistance but, for a range of frequencies near resonance (84.5 kHz), it presents an extremely high impedance. Thus, in a frequency band coincident with the resonance frequency of the tuned circuit, the ac noise current is shunted through the noise resistor (being of much lower impedance) and one can measure the voltage due to that current. A further advantage of the resonant circuit is that it eliminates the problem of shunt capacitance (detailed by Kittel[11] *et al*). Stray capacitances from the diode anode, circuit wiring, and the preamp input, are all included as part of the tuned circuit capacitance. Finally, the resonant circuit limits sensitivity to stray radiation pickup at frequencies other than that at which noise is being measured. The resonance frequency is well below the usual broadcast frequencies but high enough so that $1/f$ noise is not a concern. Alone, the resonant circuit has a $Q = 53$. It is considerably less in the circuit.



Shot noise circuit

The circuit diagram also shows a π-filter in series with the +200 V supply. It's purpose is to eliminate parasitic oscillations. The entire shot-noise circuit is housed in an aluminum box (for RF shielding).

Separate external power supplies provide the filament and anode voltages for the noise diode. **Be sure to observe polarities on the connectors**. The anode voltage should be set to +200 V on the Keithley SourceMeter. The anode current is controlled by varying the temperature of the filament (cathode). Since the electron emission current from the filament is an exponential function of filament temperature, it is important that the filament heating current be well regulated — operate the E3610A supply in the current control mode. **Make sure the filament current is turned to zero before switching the power supply on or off**. The minimum filament heating voltage/current for electron emission is about 1.9 V/0.95 A. About 4.2 V/1.42 A will produce a 10 mA diode current. You'll be operating the filament power supply in this range for the shot-noise measurements. **Do not exceed 10 mA diode current**.

Low-noise preamplifier

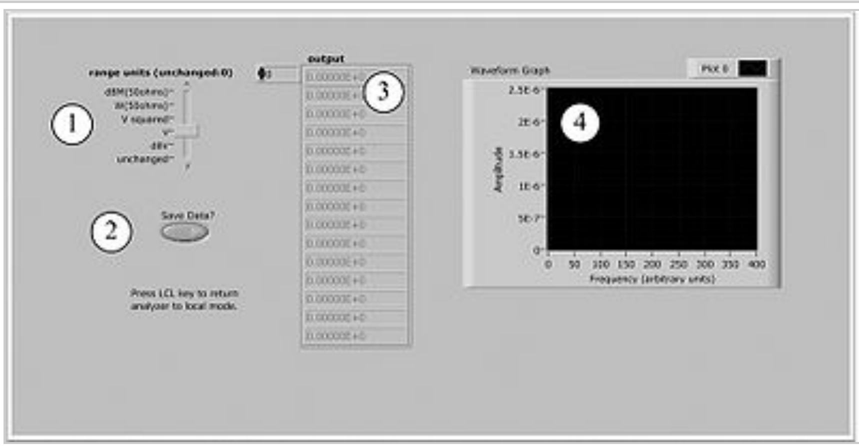
The SR560 (manual (http://www.thinksrs.com/downloads/PDFs/Manuals/SR560m.pdf#search=SR560noisefiguredata)) is a low-noise $(< 4nV/\sqrt{Hz})$ preamp with gain adjustable up to 50,000. It has a high impedance (100 MΩ + 25 pF) differential input, 1 MHz frequency response, as well as high and low frequency roll-offs. To minimize pickup of noise other than shot and thermal, the battery-powered preamp is contained in the same large RF shield enclosure (stainless steel oven) as the two noise generator boxes. Read the instrument manual carefully to fully understand its operation and controls. Choosing the proper settings will be crucial to the experiment. Noise contours are given on page 19 of the manual and need to be taken into consideration in the analysis of your data.

Spectrum analyzer

The HP3561A spectrum analyzer (manual (http://cp.literature.agilent.com/litweb/pdf/03561-90002.pdf)) is an excellent device for viewing white noise — you can see the amplified signal from the preamp and readily determine that you are measuring the desired random noise, not pickup or 60-Hz hum. The rms value of white noise is approximately equal to 1/8 the peak-to-peak value taken from the oscilloscope (ignore occasional extreme peaks in estimating the peak-to-peak value)[12]. However, the more precise measurements for this experiment will be taken with a spectrum analyzer. The HP3561A analyzer produces a frequency spectrum by recording a sequence of voltage measurements (like a digital oscilloscope) and then performing a Fast Fourier Transform (FFT) on them. In monitoring the entire spectrum, you will be able to identify EMI problems and choose the region of the spectrum most appropriate for your measurements. You can manipulate the FFT in various ways to obtain the noise voltages. Although the long-term rms value of noise is a constant, the instantaneous amplitude is totally random. Two manipulation features will be particularly useful: band analysis and spectrum averaging. For the same level of accuracy, narrowband measurements require a longer averaging time than do wideband measurements. You will need to read the manual to take full advantage of the analyzer's features. In the process you will gain much insight into the practical applications of Fourier transforms.

LabVIEW VI

You can either extract rms noise values directly from the analyzer using the cursors, or transfer the data to a computer. A LabVIEW program talks to the analyzer over a GPIB interface. Before transferring data, pause the analyzer or let it complete a series of averages. Open "hp3561reader.vi." The file lives in C:\Program Files\National Instruments\LabVIEW 8.5\user.lib. There should be a shortcut on the Desktop.



LabVIEW VI for noise measurement

- Control #1 selects the units for spectral data transferred from the analyzer.
- The "Save Data?" switch has to be ON (default) to save data to a text file. When you run the program, a dialog box asks for a file name. Use a .txt extension explicitly. The program will transfer data and write a text file containing a single column of numbers. These are the spectrum data points in the analyzer's 400 frequency bins. You can create a

corresponding frequency column later if you know the START: and STOP: frequency values displayed on the analyzer.
- Indicator #3 is the array indicator showing the spectrum data.
- Indicator #4 is a plot of the spectrum data. It should appear the same as the plot on the analyzer.

To return the analyzer to local control (front panel keys enabled), press the LCL key.

Experimental procedure

Ultimately, the aim of your measurements is to investigate the shot and thermal noise as predicted by the Schottky and Nyquist formulas and determine the values of e and k in the process. If time permits, you can try your hand at noise thermometry (thermal noise temperature dependence) by immersing external resistors in liquid nitrogen and other known temperature baths.

Use the RF signal generator, oscilloscope, etc. to calibrate and become familiar with the measurement apparatus. Refer to the operating manuals of the various instruments for details in their use and precautions. It is important to note input and output impedances of the various instruments and understand how matches (and mismatches) affect calibration. For example, the output of the SR560 is meant to drive high impedance loads and the instrument's gain is calibrated accordingly. When driving a 50 Ω load via the 50 Ω output, the gain of the amplifier is reduced by a factor of two.

Measure thermal noise first. Begin by inspecting and identifying the insides of the noise box — be sure the cover is replaced before starting any readings (it shields the apparatus). Measure noise levels (fractions of μV) for the various resistors. Make necessary corrections for amplifier noise, etc. Present your final data as a plot of the thermal-noise voltage squared versus resistance and determine Boltzmann's constant.

Follow with the shot noise measurements. Again, begin by inspecting and identifying the insides of the diode noise source box. Read the manuals for the two power supplies and understand fully how to operate them. Make certain that the preamp is not being overloaded (you will most likely have to use a small gain setting). Your measurements should be performed close to the resonant frequency of the tank circuit. Measure noise levels (a few µV) as a function of diode plate current using two or three different source resistors (notice that the tank circuit resonance is more evident when using the larger source resistors). Make necessary corrections for amplifier noise, source resistor thermal noise, etc. Present your final data as a plot of the shot-noise voltage squared versus diode current and determine the charge of the electron.

References and Notes

1. ↑ H. Nyquist, "Thermal Agitation of Electric Charge in Conductors," Phys Rev **32**, 110-113, (1928)
2. ↑ The bandwidth of an instrument is usually defined as the frequency span between the –3dB points of the instrument's band-pass function. The equivalent noise bandwidth (ENB) is defined as the frequency width of an ideal band-pass box whose area is equal to the area of the actual response of the instrument (which has a smooth roll-off). The roll-off depends on the filter window of the instrument and one must calculate the ENB from the response function (the shape of the window). The nice people at Hewlett Packard have done this for you and have tabulated the ENB for various windows in appendix E of the HP3561A manual (http://cp.literature.agilent.com/litweb/pdf/03561-90002.pdf) .
3. ↑ J.B. Johnson, "Thermal Agitation of Electricity in Conductors," Phys Rev **32**, 97-109 (1928)
4. ↑ A noise diode is operated with its anode voltage large enough to collect all the electrons emitted by the cathode, hence the name temperature-limited diode. In a diode operated in the space charge limited region, the current of electrons is not completely random, since the motion of each electron is influenced

by the electrons already present in the space charge; the shot noise is consequently decreased by a factor, the *space charge depression factor*, which is difficult to calculate. This is why a temperature-limited diode is used in the experiment.
5. ↑ W. Schottky, "Über spontane Stromschwankungen in vershieden Elektrizitätsleitern," Ann. der Phys. **57**, 541-567 (1918)
6. ↑ This noise arises from resistance fluctuations in a current carrying resistor (or any other electronic component) and the mean squared noise voltage due to 1 / *f* noise is given by *** , where *A* is a dimensionless constant (10^{-11} for carbon), *R* is the resistance, *I* the current, Δf the bandwidth of the detector, and *f* is the frequency to which the detector is tuned.
7. ↑ A.W. Hull and N.H. Williams, "Determination of Elementary Charge e from Measurements of Shot-Effect," Phys Rev **25**, 147-173 (1925)
8. ↑ P. Horowitz and W. Hill, *The Art of Electronics*, 2nd ed (Cambridge University Press, 1989), Chapter 7: Precision Circuits and Low-Noise Techniques
9. ↑ The shot noise measurement will necessarily also include the thermal noise of the source resistor, but the two are uncorrelated and the shot noise can be adjusted to be much greater than the thermal noise.
10. ↑ L. Stigmark, "A Precise determination of the charge of the electron from shot-noise," Arkiv für Fysik **5**, 399-426 (1952)
11. ↑ P. Kittel, W.R. Hackleman, and R.J. Donnelly, "Undergraduate experiment on noise thermometry," Am. J. Phys. **46**, 94-100 (1978)
12. ↑ H.W. Ott, *Noise Reduction Techniques in Electronic Systems*, (Wiley-Interscience, 1976)

Additional Reading

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- A. van der Ziel, Noise In Solid State Devices and Circuits, (John Wiley & Sons, 1986)
- A. van der Ziel, Noise In Measurements, (John Wiley & Sons, 1976)

Papers (copies are in the Bench Notes):

- A.W. Hull and N.H. Williams, "Determination of Elementary Charge e from Measurements of Shot-Effect (http://www.fas.harvard.edu/~phys191r/References/a1/hull1925.pdf) ," Phys Rev 25, 147-173 (1925)
- J.B. Johnson, "Thermal Agitation of Electricity in Conductors (http://www.fas.harvard.edu/~phys191r/References/a1/johnson1928.pdf) ," Phys Rev 32, 97-109 (1928)
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- Y. Kraftmakher, “A shot-noise experiment with computer control and data acquisition,” Am J Phys 73, 984-986 (2005)
- D.R. Spiegel and R.J. Helmer, "Shot-noise measurements of the electron charge: An undergraduate experiment," Am J Phys 63, 554-560 (1995)
- W.T. Vetterling and M. Andelman, "Comments on: Undergraduate experiment on noise thermometry," Am J Phys 47, 382-384 (1979). This is an older version of our own experiment.

Bench Notes

- Hewlett Packard 3561a Dynamic Signal Analyzer (http://www.fas.harvard.edu/~phys191r/Bench_Notes/A1/hp3561a.pdf)
- Stanford Research Systems SR560 Voltage Preamp (http://www.fas.harvard.edu/~phys191r/Bench_Notes/A1/sr560.pdf)
- Keithley 2400 SourceMeter Quick Results Guide (http://www.fas.harvard.edu/~phys191r/Bench_Notes/A1/2400_quick.pdf)
- Keithley 169 Digital Multimeter (http://www.fas.harvard.edu/~phys191r/Bench_Notes/A1/169.pdf)
- Agilent E3610A DC Power Supply (http://www.fas.harvard.edu/~phys191r/Bench_Notes/A1/agilent_e3610a.pdf)
- Sylvania Noise Diode (http://www.fas.harvard.edu/~phys191r/Bench_Notes/A1/NoiseDiode.pdf)
- RF Signal Generator (http://www.fas.harvard.edu/~phys191r/Bench_Notes/A1/RFgen.pdf)