B-5 Relativistic Mass

From Physics 191r

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RELATIVISTIC MASS OF THE ELECTRON

first experiment: II

Contents

LEARNING GOALS

- Measure the mass of electrons traveling at relativistic speeds
- Become familiar with vacuum techniques
- Work safely with high voltages
- Learn how a Geiger tube works and how to operate one
- Perform nonlinear curve fits

INTRODUCTION

In this experiment the momentum and velocity of high-energy electrons are measured to obtain the dependence of the mass-to-charge ratio on velocity, for electrons of energy from 600 keV to 900 keV. Assuming that the electron charge is independent of velocity, we wish to measure the mass of the electron, which appears to change with velocity.

In a book written for a general audience, Einstein describes the effect in terms of inertial mass:

A body at rest has mass but no kinetic energy, that is, energy of motion. A moving body has both mass and kinetic energy. It resists change of velocity more strongly than the resting body. It seems as though the kinetic energy of the moving body increases its resistance. If two bodies have the same rest mass, the one with the greater kinetic energy resists the action of an external force more strongly.[1]

In his commentary on this quotation, Carl Adler writes:

… I wish to address the use of the word "seems" in the quoted passage. This word is well advised because the apparent increase in "resistance" with velocity of a moving particle is an illusion. It comes about simply because of time dilation. An applied force on a moving particle of a given rest mass apparently takes a longer time to accelerate it when the particle is moving faster than the same force applied to the same particle when moving slowly. Thus the particle appears to have more resistance to acceleration at high speeds. In reality, measured by a clock instantaneously traveling with the particle, the same force always produces the same effect in the same time interval.[2]

Instead of defining a "relativistic mass," we consider an electron of mass m and charge e moving perpendicular to a uniform magnetic field, B. The momentum of such an electron is $p = eBR$ where R is the radius of curvature of the electron's trajectory. In our spectrometer, a slit is used to define the radius, which selects electrons of a given momentum for fixed magnetic field.

Electrons, which pass perpendicularly through the slit, enter a region between two parallel plates. For a certain magnitude and sign of the electric field, the electric and magnetic forces on the particle are equal and opposite:

$$e\vec{E} = e\vec{v} \times \vec{B}.$$

Therefore,

$$v = \frac{E}{B}.$$

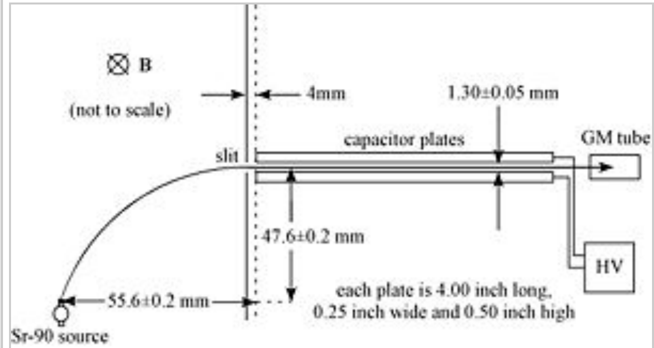
Measuring p and v independently, one can verify the relativistic expression

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}.$$

INSTRUMENTATION

Spectrometer

A schematic diagram of the spectrometer is shown at right. Refer also to photographs on the course web site and in the bench notes.



Beta spectrometer schematic.

For a given electron energy, the magnetic field (into the page), determines the radius of curvature of the electrons' trajectory. The electrons that enter the slit with velocity vector parallel to the capacitor axis can be detected when the electric force is equal and opposite to the magnetic force. The source position is fixed and its location relative to the end of the capacitor (not the slit) is shown in the figure. The magnetic field is fixed, and the electric field is swept through a suitable range. The count rate from the Geiger-Mueller tube is measured as a function of time. A peak in the count rate appears when the electric force balances the magnetic force.

If you look at the bottom of the vacuum chamber you will notice a thin brass bar screwed onto it. This bar serves as a "stop" -- that is, the vacuum chamber should be pushed into the magnet as far as it will go; the "stop" prevents you from pushing it in too far and properly positions the chamber within the magnet.

Electron source

The electron source is strontium-90. The activity is 370 MBq as of September 2003. ^{90}Sr has a half-life of 29 years, and a maximum beta energy of 0.54MeV. Its daughter, yttrium-90 is also a beta source. Its half-life is 2.7 days, and maximum beta energy is 2.3 MeV. Due to the high energy of the beta particles, the brass body of the spectrometer transmits radiation. The count rate at the surface of the spectrometer directly above the source is greater than 50 mR/hr.

To minimize radiation exposure, avoid opening the spectrometer. Geometric parameters are needed for the analysis, and nominal values are given in the diagram above. If opening the spectrometer is necessary, proper precautions must be taken. Consult the faculty or staff before opening the spectrometer.

The range of energies accessible to the apparatus is roughly 0.5 MeV to 0.9 MeV. The ultimate vacuum of the spectrometer and surface roughness of the capacitor determine the maximum energy detectable. The capacitor tends to discharge at high voltages. If the spectrometer is opened, please do not touch the plates or insert feeler gauges or other measuring tools into the gap. The plates are made of soft copper and scratch easily. Scratches cause arcing between the plates.

Geiger-Mueller Detector

The electrons leave the evacuated spectrometer through a thin aluminized mylar window and travel through ~ cm of air to the detector. The detector[3] is a Geiger-Mueller tube (GM detector) with a very thin mica end window. The areal density of the GM detector window is 1.5 – 2.0 mg/cm^2. Be careful not to accidentally puncture either window; both windows are very fragile. Read about the care and feeding of GM detectors elsewhere (see Knoll or Moore/Davis/Coplan, for example). The general operating procedure is as follows: With the high voltage at zero, connect the GM detector to the MHV connector on the rear panel of the HV supply/scaler[4]. Place a radioactive source (preferably a β emitter) near the GM tube. Depress RESET and then COUNT. Slowly increase the high voltage until counting just begins (threshold voltage). Then determine the plateau region where the GM tube's efficiency is "flat" with respect to high voltage. Caution: maximum voltage should be about +400 V -- do not destroy the GM tube by application of excessive voltage. Note also that the GM tube, approaching the end of its life, may become erratic and change its rate drastically up or down under otherwise constant operating conditions. The sensitivity may vary with count rate even for a good GM tube. The GM counter output (BNC connector on rear panel) is a TTL pulse and this signal goes to the PCI-6014 I/O card input.

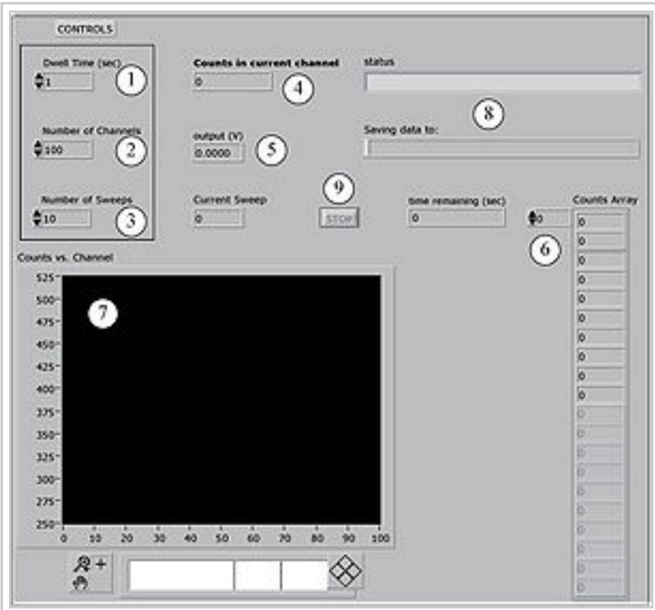
Magnetic field

A Hewlett Packard 6114A Precision Power Supply, wired for external constant-current resistance programming, powers the electromagnet. The programming resistor, (in a small aluminum box) determines the output current. The front-panel controls can override the programming resistor, so they should be set well above the expected operating parameters. A 1 Ω resistor is wired in series with the magnet. The voltage across this resistor serves as a current monitor.

A digital teslameter[5] is used to determine magnetic field strengths. It is a microprocessor based Hall-effect instrument. The probe is very expensive, so please handle the instrument with care and follow the instructions in the manual to achieve high precision measurements.

Data acquisition, control and LabVIEW vi

We desire to measure the count rate from the GM tube as a function of time. A Multichannel Scaler (MCS) accomplishes this task. It stores a histogram of counts versus time. The MCS consecutively advances through channels, "dwelling" in each channel for a preset amount of time. It stores the number of counts received during the dwell time in that particular channel. Advancing through all the channels once is referred to as a "sweep." The MCS can be set to execute many sweeps, adding to data taken during previous sweeps. By noting the plate voltages at the beginning and end of the ramp and assuming a linear ramp (How good of an assumption is this?), one can convert the counts vs. time histogram to counts vs. electric field and ultimately, electron velocity. Be aware of limitations in high voltage slew rates when setting the dwell time.



Relativistic mass LabVIEW multichannel scaler.

The MCS is a computer-based "virtual" instrument. A LabVIEW program called mcsrelmass.vi communicates with a National Instruments PCI-6014 Multifunction I/O card. A counter input on the card accepts pulses from the Geiger Tube Controller (rear-panel BNC jack). The MCS input is connected to a BNC jack labeled “PCI-6014 IN” next to the vacuum pump power switches. The LabVIEW program in conjunction with the PCI-6014 also supplies a 0 – 5 Volt analog output to program the high voltage power supplies. The output is a step function where each channel corresponds to a voltage step. The analog output BNC jack labeled “PCI-6014 OUT” is next to the vacuum pump power switches. See the diagram below for more information.

- (1). Dwell Time control determines the amount of time to count at each voltage value.
- (2). Number of Channels control sets the number of voltage steps i.e., the resolution.
- (3). Number of Sweeps control specifies the number of times to repeat the voltage scan.
- (4). Counts indicator displays the number of counts in the current channel.
- (5). Output indicator gives the current value of the analog output.
- (6). Counts Array indicator shows the number of counts accumulated so far in each channel.
- (7). The graph indicates the trend in count rate as a function of channel number.
- (8). The status indicator identifies the current point in the program's procedure. One of the failure modes of the experiment is to use a high voltage value that causes sparking between the capacitor plates. The program allows time to check for this by ramping up to the maximum voltage and dwelling there for 30 seconds. During this time, check the neon bulbs mounted near the HV divider box. The bulbs are connected across one of the 1 MΩ series resistors. If the capacitor draws current in the form of a spark, a momentary voltage drop across the series resistor lights the bulbs. The program then ramps back down to zero, and begins the sweep after a suitable interval. The Saving Data To: indicator displays the path of the data file. Data is saved after each sweep.
- (9). Use the STOP button to terminate the program before completion of the full number of sweeps. Data from the current sweep is lost.

High voltage sweep and measurement

A fixed magnetic field is used to select electrons of a given momentum (actually, a small range of momenta). By varying the electric field over some appropriate range, the electron detector will see a maximum signal when the electric and magnetic forces are balanced. The electric field is varied by "sweeping" the high voltage on the deflection plates in the velocity selector. In this case, sweeping means changing the voltage on the plates linearly and symmetrically with respect to ground over some predetermined range as a function of time (a sawtooth function).

Equal and opposite voltages are applied to the two capacitor plates. Two voltage-programmable Bertan power supplies provide the high voltage. Each supply has an output range of 0 to 10 kV, one positive and the other negative. The desired program input voltage to the HV supplies is 0 to 5 volts. The PCI-6014 has an analog output used for this purpose. Use a voltmeter to check that this voltage steps between 0 and 5 volts as the multichannel scaler goes through one sweep. It is not desirable to sweep the full 20 kV[6]. Two ten-turn potentiometers are installed on a panel in front of the HV supplies. The dial labeled "ramp offset" adds a dc level to the sawtooth input and the "ramp voltage" dial attenuates the sawtooth, so any desired voltage range can be swept. The dials read directly in kilovolts and **the sum of the two numbers should never exceed 8**! A high voltage monitor output is provided for convenience but is of limited accuracy (0.1% of reading + 0.1% of maximum).

The high voltage measurements are performed with a voltage divider, which, in turn, must be calibrated using the high-accuracy (0.03%) 100 kV divider[7]. Faculty or staff should be consulted first concerning proper safety precautions and procedures, and the following pointers are given here as a reminder. First, before disconnecting any high voltage leads turn off the high voltage power supplies! Next, disconnect only one of the two spectrometer capacitor plates from the home-made voltage divider. Connect the donut-shaped dome (via the special high-voltage test lead and SHV connector) of the high-accuracy divider to the point where you disconnected the capacitor plate. Make very sure that the high-accuracy divider is properly grounded. The jack labeled "LOW" should be attached to a reliable ground such as the brass bar connected to a water pipe. Make sure that you do not touch or accidentally brush against the donut-shaped dome — it will be at high voltage when the power supplies are on! Use a digital voltmeter to measure the voltage across the resistor at the low-end of the divider chain; that is, measure the voltage between ground and the jack labeled "1V". Simultaneously monitor the low voltage output of the homemade voltage divider and calibrate that voltage against the high-accuracy divider. Turn off the high voltage power supplies before repeating the measurement process for the other polarity (the other capacitor plate).

Vacuum system

Electrons don't travel very far through the air. For this reason the spectrometer needs to be evacuated. In addition to the plumbing, the vacuum system consists of a mechanical pump, turbomolecular (http://en.wikipedia.org/wiki/Turbomolecular_pump) pump, and vacuum gauges (thermocouple and cold cathode). Consult the references below for background information on how these pieces of apparatus work. The mechanical pump is used for "roughing out" the system to bring it down to the tens of microns (one micron = 10^{-3} mm Hg = millitorr) pressure range and is referred to as a "roughing pump" in this context. It also serves as a backing pump for the turbomolecular pump and is then called a "fore pump" or "backing pump". The thermocouple gauge is good for measuring pressures down to micron range (typical mechanical pump pressures) while the cold cathode gauge is used for lower pressures achieved with the diffusion pump, typically 10^{-5} mm Hg. Consult the faculty or staff concerning procedures in "pumping down" and "breaking vacuum" after reading the appendix on vacuum procedures.

EXPERIMENTAL PROCEDURE

- Pump the spectrometer down into the 10^{-4} Torr range.
- Measure the geiger tube plateau using a test source.
- Calibrate the high voltage.
- Set up the high voltage sweep and familiarize yourself with the LabVIEW vi.
- Predict the magnetic field and electric field values for an electron in the 600 - 700 keV range.
- Set the magnetic field and take data for the energy chosen.
- Measure successively higher energies as the vacuum drops into the 10^{-5} Torr range, always staying below 16 kV across the capacitor.

"About once every year or so, Desmond sets some time aside to give Einstein's relativity theories another go . . ."

NOTES

1. ↑ Einstein, A., The Evolution of Physics, Simon and Schuster, New York, 1938, 207.
2. ↑ Adler, C., "Does mass really depend on velocity, dad?", Am. J. Phys., 55 (8), August 1987.
3. ↑ LND type 712.
4. ↑ The Nucleus, model 500 Nuclear Scaler.
5. ↑ Group 3 model DTM-141 Digital Teslameter.
6. ↑ Field emission and secondary electron emission from the plates become much more pronounced at high

voltages resulting in a marked increase in background counts. Electrical discharges are also a problem if the vacuum is not good enough and/or the cable connector(s) are "dirty", the insulation has a nick out of it, the ground shield has not been pulled back sufficiently, or the plates have been scratched. Start your data runs at a comparatively low voltage. As the pressure decreases over days and weeks, you will be able to sweep higher voltages without sparking.

7. ↑ Hallmark Standards model PVD, 100kV Potential divider.

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H.A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, The Principle of Relativity, (Dover, New York, 1952). Original papers are translated with notes by A. Sommerfeld. Worth reading pp 35-50 for content, clarity of thought and expression, and simplicity. Also pp 69-71 - does the inertia of a body depend upon its energy content?

R.T. Weidner and R.L. Sells, Elementary Modern Physics, 3rd ed., (Allyn & Bacon, Boston, 1980). Most good modern physics textbooks will help you out - see Holton above for suggestions. This one happens to have a good comparison of classical, relativistic, and extreme relativistic curves of energy vs momentum.

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J.H. Moore, C.C. Davis, and M.A. Coplan, Building Scientific Apparatus, 2nd ed, (Addison-Wesley, Reading MA, 1989).

Jacob Shapiro, Radiation Protection - A Guide for Scientists and Physicians, 2nd. ed., (Harvard Univ Press, Cambridge MA, 1981).

Vacuum techniques

S. Dushman, Scientific Foundations of Vacuum Technique, 2nd ed., J.M. Lafferty, editor, (John Wiley & Sons, New York, 1962). This is the bible on the subject and will explain everything to you.

J. Moore, C. Davis, and M. Coplan, Building Scientific Apparatus - A Practical Guide to Design and Construction, 2nd ed., (Addison-Wesley, Reading MA, 1989). An excellent book to start with. Many additional current references are contained herein.

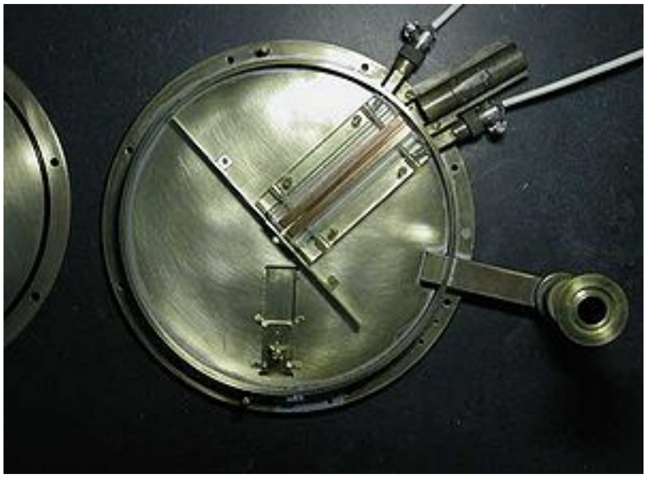
BENCH NOTES

- Bell 640 Gaussmeter (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/Bell640.pdf)
- Bertan High Voltage Source (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/BertanHV.pdf)
- Keithley 169 Multimeter (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/Keithley169.pdf)
- Keithley 179 Multimeter (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/Keithley179.pdf)
- Spectrometer: high-res tiff (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/rel_spectro1.tif)
- Spectrometer closeup: high-res tiff (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/rel_spectro2.tif)
- HP 6114A power supply (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/HP6114A.pdf)
- National Instruments PCI-6014 (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/mm0_6014.pdf)
- Canary II Dosimeter (http://www.fas.harvard.edu/~phys191r/Bench_Notes/canary.pdf)
- LND712 Geiger tube (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/b5_LND712.pdf)
- Strontium-90 beta spectrum (*Nuclear Spectrometer Applications* by Wendell H. Bradley, et. al.) (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/sr90.pdf)
- Pfeiffer Balzers TPU240 turbomolecular pump (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/tpu240.pdf)
- Pfeiffer Balzers TCP121 turbomolecular pump controller (http://www.fas.harvard.edu/~phys191r/Bench_Notes/B5/tcp120_121.pdf)
- Dell Optiplex 980 Technical Guide (http://www.fas.harvard.edu/~phys191r/Bench_Notes/optiplex-980-tech-guide.pdf)

PHOTOS

APPENDIX: Vacuum system procedures

The vacuum system incorporates two pumps, a mechanical pump located on the floor, and a Pfeiffer Balzers TPU 240 turbomolecular pump in the instrument rack next to the spectrometer. The pumps operate in series. The lowest pressure, achieved after several days of pumping, is on the order of 10^{-5} Torr.



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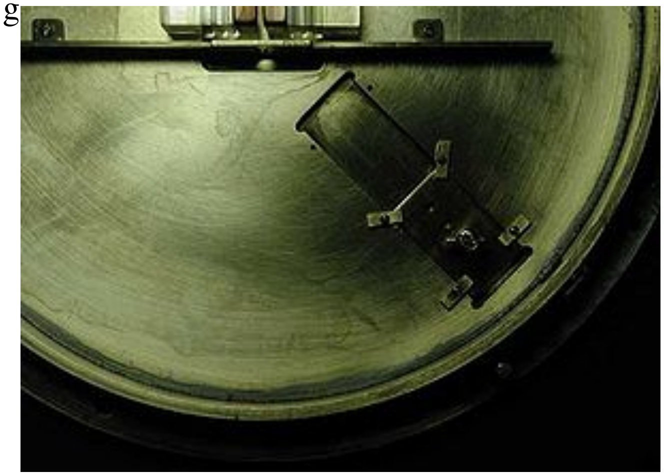

ırbo pump. The power button is on the right side of the TCP 121

controller. To prevent automatic venting, a small rubber stopper is kept in the vent valve. The turbo pump ramps up slowly over several minutes. A segmented LED indicator lights from left to right as the pump approaches its maximum speed of 60,000 rpm. The Standby and Heater controls are not used. After a few hours of pumping, the chamber should reach the 10^{-4} Torr range. DO NOT turn on the cold cathode gauge until the chamber pressure is less than 10^{-3} Torr.

To bring the chamber back to atmospheric pressure, first switch off the cold cathode gauge, then switch off the turbo pump and wait for it to coast to a stop. This may take 30 minutes or so. The pump's automatic vent valve is blocked with a small rubber stopper. Lastly, unplug the mechanical pump, and admit nitrogen gas to the chamber and vacuum system with the manual vent valve near the chamber.