

Lab 1: Measurement and Uncertainty

I. Before you come to lab...

- A. Read through the handout on the course website, Chapters 1-2 from Taylor, *An Introduction to Error Analysis*. These chapters will introduce the main ideas behind measurement and uncertainty that we will explore in this lab and throughout the semester. Although only the first two chapters are required reading, the full text is on reserve at Cabot and contains detailed explanations of the main concepts from experimental science. Other references you may find helpful:

1. B. S. Massey, *Measures in Science and Engineering*
2. S. K. Kimothi, *The Uncertainty of Measurements*

- B. Read through this handout in its entirety.

- C. Download the Logger Pro software and learn how to use it.

1. Logger Pro for Mac or Windows can be obtained from

<http://downloads.fas.harvard.edu/download>

You will need to log in with your Harvard ID and PIN. Logger Pro 3.8.2 requires Mac OS X version 10.4.11 or 10.5.1 or later, or Windows XP/Vista/7. If you have a PC or Mac but are unable to install Logger Pro, contact Joon for help. His email address is pahk@fas.harvard.edu. If you don't have a PC or a Mac of your own, you can use Logger Pro on a computer in the Science Center computer lab to complete this part.

2. To familiarize yourself with Logger Pro, install the software and then work through some of the tutorials. You



can do this by starting Logger Pro and then clicking on the  in the toolbar. The Open File dialog box will open to a folder called "Experiments." Inside that folder, find the sub-folder called "Tutorials." Here are the tutorials that you should do:

- 1 Getting Started
- 5 Manual Data Entry
- 7 Viewing Graphs
- 8 Stats, Tangent, Integral
- 9 Curve fitting

- D. Complete the pre-lab question as part of Homework #1, and turn it in with the rest of your homework by 9:30 am Tuesday.

E. On the day of the lab...

1. Come on time to Science Center 115.
2. Bring this handout.
3. Bring a writing implement.

II. Introduction

- A. This first lab is intended as a gentle introduction to the ideas of measurement and uncertainty. You'll measure the periods of 6 pendulums using a stopwatch and see how the uncertainty in the measurement depends on the number of times you repeat the measurement.

B. Learning objectives for this lab:

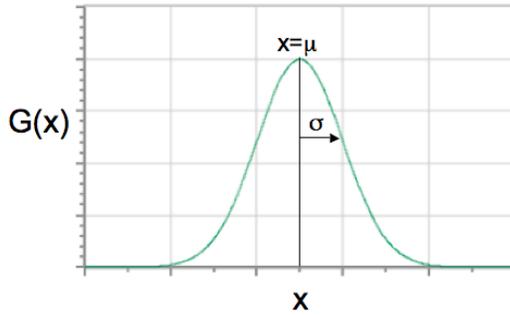
1. Understand what uncertainty is and what determines the uncertainty of a measurement
2. Understand what is learned by repeating a measurement, and be able to figure out how many times you should repeat it
3. Understand that a measurement by itself, even with uncertainty, is meaningless unless compared to a prediction or to another measurement

III. Background

(If you haven't already read the chapters from Taylor posted on the course website, do so now before continuing.) In an experimental setting it is just as important to specify not only what you know, but how well you know it. The purpose of this first lab is to introduce some of the basic ideas behind the study of uncertainty.

A. Gaussians

1. You know from reading the Taylor excerpt that all measurements are subject to some random variation, which we refer to as "uncertainty." It turns out (for reasons we will see later in the semester) that the vast majority of the time, this random variation is distributed according to a "bell curve" like the following:



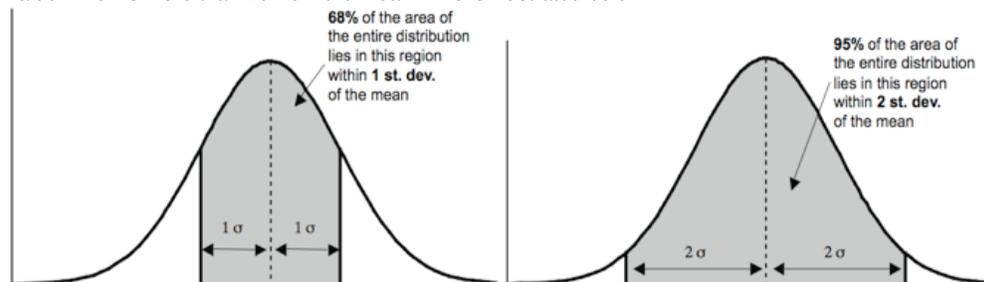
This function has lots of names, including bell curve, normal distribution, and Gaussian distribution. We'll use the name Gaussian (pronounced GOW-see-an) in this course. The Gaussian is named for K.F. Gauss, a 19th-century pioneer in math and physics.

2. We won't go into a lot of probability theory here, but the most important qualitative thing to observe about the Gaussian distribution is that it's peaked around a central value, called the **mean**. In the graph above, the value of the mean is represented by the Greek letter μ (mu). The significance of this shape is that if you repeat a measurement lots of times, most of the values will be "close" to some central value, the mean.
3. We can be a little bit more quantitative about what we mean by "close." A Gaussian distribution is characterized not only by its mean μ , but by a second parameter called the **standard deviation**, represented by the Greek letter σ (sigma). The standard deviation (which is always positive) is a measure of the width, or spread, of the distribution. The smaller σ is, the more sharply peaked the distribution. A large value of σ corresponds to a wide, flat distribution, making it harder to identify the peak.

4. Rule of 68 and 95

Using the mean and standard deviation, we can state the following two numerical rules about measurements that follow a Gaussian distribution:

- a. **68% of the time, the measurement falls between $\mu - \sigma$ and $\mu + \sigma$** , i.e. within one standard deviation of the mean. In other words, it has about a 2/3 chance of being within "one sigma" of the mean.
- b. **95% of the time, it falls between $\mu - 2\sigma$ and $\mu + 2\sigma$** . It is quite rare for a Gaussian variable to have a value which is more than 2σ from the mean. This is illustrated below:



5. When we state the value and uncertainty of a measurement, **the uncertainty corresponds to a 68% confidence interval**, by convention. You can think of this as a quantitative refinement of the general idea that we are "reasonably certain" that the measured value lies within the uncertainty range. This convention is chosen because we assume that most measurements do obey a Gaussian distribution, which means that

the uncertainty of a single measurement corresponds to one standard deviation.

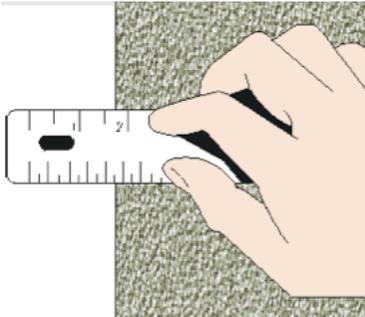
- a. For example, if I state that a certain distance is 2.77 ± 0.02 cm, that means that I think repeated independent measurements of the same distance would lie between 2.75 and 2.79 cm about two-thirds of the time.
 - b. Given what we know about Gaussians, it's essentially equivalent to claiming that the distance would be measured between 2.73 and 2.81 cm (2σ away from the mean on either side) about 95% of the time.
 - c. *Warning:* even though we define the uncertainty range as the one that corresponds to a 68% confidence interval, that doesn't mean that the uncertainty is always equal to the standard deviation. "Why not?" you ask. For the answer, read section C on Standard Deviation and Standard Error.
6. In case you're curious, the Gaussian distribution function has the following mathematical form:

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

The weird-looking factor outside of the exponential is there for normalization; most of the time, we won't care about it. Actually, most of the time, we won't really care about the functional form of the Gaussian at all, but it is useful to be able to fit a Gaussian to a histogram to estimate the mean and standard deviation of a set of measurements. When you do fit a Gaussian,

B. Uncertainty of a single measurement: Reading error

- 1. During this lab you will learn how to calculate the uncertainty of a measurement by repeating the measurement many times and generating a histogram of results. Is this the only way to determine the uncertainty? Luckily, the answer is no. For many measuring devices, you can get a very good estimate of the uncertainty of a single measurement just from common sense.
- 2. The simplest example is using a ruler to measure the length of something. Here's an example: suppose you use this ruler to measure the position of the left edge of the surface shown:



It looks like it's a bit under 1.75 inches. But how do we get the uncertainty? *We have to estimate the minimum and maximum values the reading could have without us noticing the difference.*

- a. Here's how I would do it: the upper bound seems to be 1.75" or maybe 1.74"; the lower bound is trickier, but I'd guess it's about 1.70". (The two ruler marks on either side are 1.50" and 2.00"; it's pretty clearly more than 40% but less than 50% of the way from 1.50 to 2.00.) So I would say that the position is 1.72 ± 0.02 inches; the uncertainty of 0.02" is called the *reading error* of the measurement.
 - b. Another measurer might well disagree with my estimate of the reading error, but probably not by a lot. Determination of reading error can be a little crude, but the important thing is to get a reasonable estimate of the uncertainty without having to do the measurement many times.
3. Not all reading errors are created equal. For example, using the same ruler to measure the position of a rough edge (rather than a smooth one) would give a larger reading error. Alternatively, using a ruler with more closely spaced markings might well give a smaller reading error. Ultimately it always comes down to your judgment and common sense.
4. Once you have estimated the reading error on a measurement, you can treat it just like a statistical uncertainty, as if the reading error were a standard deviation of repeated measurements. That may be a stretch, but estimating reading error is a quick and dirty approach to uncertainty that produces generally

reasonable results.

▼ C. Standard Deviation and Standard Error

1. If the data we take can be reasonably approximated by a Gaussian, then we know that about 68% of the measurements will be within 1σ of the mean, and 95% within 2σ . Therefore, when we use a *single* measurement as a guess for the mean value, the uncertainty of that guess is just σ , the standard deviation.
2. If we take repeated measurements, the *best* guess we can make for the true value is the average (mean) of all the measurements. Intuitively, it should make sense that the more times we repeat the measurement, the better this guess becomes. The uncertainty of this best estimate is related to the **standard error** (or SE) which has the formula:

$$\text{Standard Error} = \frac{\sigma}{\sqrt{N}}$$

The N in the formula is the number of repetitions that get averaged to produce our best guess. Essentially, **the SE represents the uncertainty of our best guess.**

3. Note that when $N=1$, the SE is equal to the standard deviation. This confirms our earlier statement that the uncertainty of a single measurement is equal to the standard deviation.
4. Standard error is useful for knowing how much data you *should* take in any given experiment. The more data points you take, the smaller the standard error will be. However, the formula for SE produces diminishing returns. Making a measurement ten times instead of once gives you about a factor of 3 improvement on your uncertainty. That may seem quite attractive in many cases. But if you want another factor of 3 improvement, you have to do it 100 times. To avoid unnecessary grunt work, try to go into an experiment with an idea of an acceptable tolerance for the uncertainty.

▼ D. Basic error propagation: "Worst-case" method

1. Chapter 2 of Taylor gives a few formulas for *propagation of error*, which is the process by which we can determine the uncertainty of a quantity which was not itself measured directly, but instead calculated from one or more measured values. Error propagation answers the question, "how do the uncertainties in the measured values *propagate* to the uncertainty of the final answer?"
2. The simplest way to answer this question is also the crudest, but it has the virtue of being quite straightforward to understand and apply: assume the worst-case scenario in each direction, and look at the range of values you get for a final answer.
 - a. For example, suppose I have measured the values $A = 22 \pm 3$ and $B = 15 \pm 2$, and I am interested in the quantity $C = A - B$. The best-guess value of C is just the mean value of A minus the mean value of B , which is $22 - 15 = 7$. Now I can estimate the uncertainty by looking at the possible values of A and B that will make C as *big* as possible, and as *small* as possible. To make C as big as possible, A needs to be as big as possible (25) and B needs to be as small as possible (13), so C could be as big as $25 - 13 = 12$. Similarly, C could be as small as $19 - 17 = 2$. The range of values of C is from 2 to 12, so the "worst-case" uncertainty for C is 5, and I would say that $C = 7 \pm 5$.
 - b. No matter how complicated the expression is for the final quantity whose uncertainty we want to know, this simple process will do the trick, although sometimes you will have to think carefully about whether you "want" A to be big or small in a given expression in order to make C big or small. The equations for the uncertainty of a difference (2.18) and uncertainty of a product (2.28) given in Taylor were derived from this method too. If you are comfortable with those equations, you can use them instead of applying the method directly. The two are entirely equivalent, but applying the formulas is probably faster if you understand how.
 - c. As you might imagine, the worst-case method tends to *overestimate* the actual uncertainty of the final quantity. (If A and B have independent random errors, then it'd be pretty unlucky to have A be as large as possible at the same time that B is as small as possible.) In a later lab, we'll learn some more complicated methods that can give us a more accurate and smaller (and thus more useful) answer for the probable uncertainty in error propagation. But for now, the worst-case method is preferable because it is easy to understand and use, and it does give results that are quite close to the best answer.

IV. Materials

A. Pendulum

B. Digital stopwatch



- 1.
2. Press the right button to start or stop the timer.
3. Press the left button to reset the timer to zero.

C. LabPro interface



- 1.
2. The LabPro interface is the green box that allows you to connect a wide variety of different sensors to the computer via the Logger Pro software. Today, you'll be using it with the photogate.

D. Photogate



- 1.
2. The photogate connects via the digital channels ("Dig/Sonic") to the LabPro interface. Logger Pro should recognize the photogate upon connection.
3. A beam of infrared light from one arm of the photogate is directed to a detector on the other arm. When this beam is blocked (e.g., by a hand or, say, a pendulum bob), a change in signal will be registered.

V. Procedure

A. Before you begin...

1. **Open the Photo Booth application and take a picture of your lab group. Drag it into the space below:**
2. **Type in your names (from left to right in the above photo):**
3. Make sure you answer all of the questions in the boxes below. (Even though they are bold-faced, it's easy to gloss over some of them.) If you are unsure of what you are being asked to do/answer, ask your TF.

B. Measuring one period, numerous times—Stopwatch

In the first part of the lab, you will attempt to determine the periods of different pendulums. We are not as interested in the period itself as we are in the *uncertainty* in the determination of the period. Our **goals** are to understand what this uncertainty means and how to estimate it.

1. Each member of your group should get a stopwatch and a piece of paper. You will each measure the period

of the pendulum several times. Before measuring anything, spend some time talking amongst your group about the best way of measuring the period. Once you have all decided upon a method, stay consistent for all of the stopwatch measurements you take today. **Write a short description of your timing method here:**

2. Your TF will assign your group to one of six pendulums in the lab. **Measure the length of the pendulum, including uncertainty due to reading error.** Don't forget to include units on all of your measurements and uncertainties.

length=
uncertainty=

3. Time a single period of the pendulum using the stopwatch. Again, make sure you are all using the same technique to time the pendulum.
4. You don't need to estimate the reading error of the stopwatch measurement, because we won't be using it. **Why not? What's the difference between the stopwatch measurement and the ruler measurement?**
5. Record the measured period on paper and repeat the procedure until each group member has measured the period ten separate times.
6. Open the Logger Pro file "Lab1.cmb1" and enter all of the results recorded by your group in the data table, under the column labeled **TS**. If there are three of you in the group, there should be a total of 30 entries in the column. (If the program prompts you about the sensor setup, click on "Use File As Is.")
7. Create a histogram of the column **TS**. Try to make a reasonable choice for the bin width. You don't want most of the bins to be empty or have only one data point, but neither do you want the bins to be so wide that almost all of the values are in just one or two bins.
8. From the histogram, click on the  Statistics button to **calculate the statistical mean and standard deviation of your 30 data points.**

mean =
std deviation =

9. Try fitting a Gaussian to the histogram by clicking the  Curve Fit button. In the box for "General Equation," scroll to the bottom and select PS2 Gaussian. This function has three parameters: the amplitude A (which we don't care about), the mean M, and the standard deviation S. Before continuing, make sure that the Gaussian curve drawn by Logger Pro **reasonably** describes the data in your histogram. You are much smarter than the computer—every time you fit a curve to some data, you should check that the curve looks sensible.

What are the mean and standard deviation of the best-fit Gaussian? (Note: you might find that the program calculates a best-fit value of S which is negative. If so, ignore the minus sign; only S^2 enters into the equation for the Gaussian, anyway.)

mean =
std deviation =

- 10. If these values are not similar to the statistical mean and standard deviation you previously calculate, or if you are having other problems getting a Gaussian fit, you may need to adjust the bin settings for your histogram.

- 11. **Paste a screenshot of your histogram and Gaussian fit in the space below:**

- 12. Based on these 30 data points, **what is your best estimate of the pendulum period? What is the uncertainty of this estimate?** Don't forget to specify the correct units in both of your answers.

period =
uncertainty =

- 13. Above, you calculated the uncertainty in your period measurement via LoggerPro (forming histograms, fitting the data to a Gaussian, etc.). This process, although not very difficult, is not very intuitive. We can estimate a "worst-case" standard deviation by taking half of the *range* of our data, where the range R is defined as R = (largest data point) – (smallest data point). Similarly, we can estimate the standard error by

$$SE \approx \frac{R}{2\sqrt{N}}$$

How does this estimated uncertainty compare to your calculated uncertainty above? Does this seem like a reasonable estimate?

For the remainder of the lab, feel free to use the estimated value for uncertainty. Having an intuitive "worst-case" value for uncertainty is better than having no value at all.

- 14. **What are the possible sources of uncertainty in your time measurements?**

Which of these contributes the most to the overall uncertainty in your time measurements?

- 15. Save your work in both NoteBook and Logger Pro before moving on to the next part.

C. Measuring one period, numerous times—Photogate

In this part of the lab, you will measure the period of the same pendulum using a more sophisticated timing apparatus. Our **goals** are to see what effect the measuring apparatus can have on both the value and the uncertainty of a measurement, and to see in practice what it means to compare two measurements to each other.

- 1. With the pendulum at rest, position the photogate so that the bob rests between the two arms of the photogate.
- 2. When you are ready to collect data, set the pendulum into motion and click the  Start button to collect data.
- 3. Data collection will run for about 1.5 minutes. Make sure you have collected data for 30 periods.
- 4. As before, **make a histogram and use the**  **Statistics button to determine the mean and standard deviation.**

mean=

intercept=



- f. A pendulum can be used to calculate the value of g , the acceleration due to gravity, because an ideal

$$T^2 = \frac{4\pi^2}{g} L$$

pendulum with length L and period T obeys the equation $T^2 = \frac{4\pi^2}{g} L$. Using your plot of T^2 vs. L , calculate a value for g with uncertainty. (Hint: you will need to do some error propagation.)

- g. Compare your value of g to the accepted value (9.81 m/s^2).
- h. What are some advantages of calculating g from your line of best fit instead of just calculating it from one data point?

▼ VI. Conclusion

- A. Congratulations! You've reached the end of the first lab.
- B. Please take 1 minute to fill out a survey on your lab experience:
<http://www.zoomerang.com/Survey/WEB22B65HE44LP>
- C. There is a helpful sheet explaining how to submit your lab report at each station. Read and follow the directions carefully to ensure that your lab report is properly submitted.

▼ VII. Pre-Lab Assignment from the Problem Set

(These questions are here for reference.) *Before doing this problem*, read through the lab handout (from the course website) and the supplementary reading from Taylor. Then answer the following questions.

- A. In lecture on Tuesday, each of you measured the distance from the Science Center to Widener Library from a satellite map. Describe your measurement technique in a sentence or two. What numerical result did you get? Estimate the *reading error* on your measurement.

All of the values submitted by the students were recorded and entered into a Logger Pro file, which you can download from the course website. (By the time you get here, you should have already downloaded Logger Pro and completed the recommended tutorials.) Save the file to your computer and then open it in Logger Pro. The data will be there in a column labeled "Distance."

- B. Make a histogram of the values by going to the Insert menu → Additional Graphs → Histogram. Then double-click on the histogram to bring up the graph options, and then play around with the Bin Size. A bin size of 1 is probably too small (most bins are empty or contain only one point); 100 would certainly be too big (all of the points fall into 1 or 2 bins). Is there a bin size that makes the histogram look "smooth"? Does the histogram look approximately like a Gaussian (normal distribution)? Click on the Stats button in the toolbar to calculate the mean and standard deviation of the data. What are they?
- C. Recall what you learned about repeated measurements from the lab handout. From the entire data set, what is your new best guess for the distance measured? Calculate the *uncertainty* of this guess. The uncertainty should be *smaller* than the standard deviation of the data. Why?