Lab 2: Background and Useful Information

Objective

- In this lab, you'll learn familiarity with circuits by solving two "puzzles" involving measuring some unknown quantities. The key to solving these puzzles will be a good understanding of circuits, both on a theoretical and a practical basis. You will also investigate the behavior of an RC circuit as a function of time.

Background

- In this lab, you will be exploring the behavior of electrical components connected in circuits. The most basic thing to keep in mind is that nothing interesting will happen at all unless there is a *circuit*—that is, a closed loop where charge can flow.

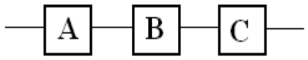
- The two major concepts of circuits are current and voltage.
  - ***Current*** (abbreviated i) is the rate of flow of charge.
    - It is measured in amperes (A), or amps for short. An amp is a coulomb per second. That is a very large current (remember how much charge 1 coulomb is), so in practice we will often be dealing with milliamps (mA) and microamps (μA).
    - In a simple circuit consisting of one loop, current flows continuously—every circuit element in the loop has the same current flowing "through" it.
    - The preposition "through" is a very useful memory aid—if you think of current as something that goes *through* a circuit element, then it makes perfect sense that the same current also goes through the next element in the loop. It's only at junctions that current divides or combines.

  - ***Voltage*** (ΔV) is another name for potential difference. Voltage is a measure of how much work it would take to move a unit charge from one place to another.
    - It is measured in volts (abbreviated V).
    - Remember that potential is something which is a function of position, so a potential *difference* is something that depends on *two* positions or two points on a circuit. We will often speak in terms like "the voltage drop from A to B" (which means "the potential at point B minus the potential at point A") or "the voltage across the resistor R" (which means "the potential at one end of the resistor minus the potential at the other end").
    - Voltages are considered to be "across" rather than "through" circuit elements. If you have two resistors in a row, they do not, in general, have the same voltage across them. However, if you add up all of the changes in voltage as you go around a closed loop, you must get a total of zero when you return to your original position because the final potential is equal to the initial potential.

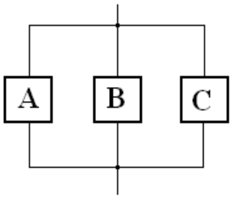
- **Ohm's law**
  - In many circumstances, the current through and voltage across a circuit element are proportional to each other. This empirical fact is known as Ohm's law.
  - The proportionality constant between ΔV and i is called the resistance, R. Ohm's law can be stated quantitatively as: $\Delta V = iR$.
  - Resistance is measured in ohms (Ω). An ohm is a volt per ampere. Because amps are so large, useful resistances are often on the order of kiloohms (kΩ) or megaohms (MΩ).
  - All real circuit elements have some resistance. However, when the circuit contains resistors or light bulbs, the comparatively small resistance of other circuit elements such as wires can often be neglected.

Series and parallel

- Two or more devices which are in *series* are connected in the same branch. Therefore they have the same current passing through them (but not necessarily the same voltage across them).



- Two or more devices which are in *parallel* are connected independently between the same two points. Therefore they have equal voltages across them (but not necessarily the same current through them).



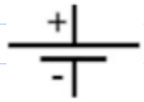
Power

- The electrical power (rate of change of electrical energy per unit time) of a device is equal to the current through it multiplied by the voltage across it: $P = i\Delta V$. Electrical power is measured in watts (1 W = 1 Joule/second), just like mechanical power.
- Being very careful with the sign of electrical power leads to some useful insights:
  - When the potential *decreases* in the direction that current is flowing, ΔV is negative, so that P is also negative. That tells us that electrical energy is being *lost* in that circuit element. This is always the case with resistors.
  - When the potential *increases* in the direction that current is flowing, ΔV is positive, so P is also positive. This is usually the case with a battery.
- For resistive elements (resistors, light bulbs, heaters, etc.), the power is the rate at which electrical energy is being converted into heat or light. We often talk about power "dissipated" in a resistor or bulb; as a result, the minus sign is often dropped with the implicit understanding that electrical energy is being lost or dissipated.
- For a battery, P is the rate at which the battery is *supplying* electrical energy, which the rest of the circuit can use to do work. It is also the rate at which the chemical energy stored in the battery is being depleted.

Common circuit elements

Battery

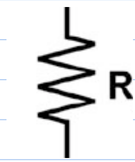
- A battery can be considered as a source of constant voltage. The voltage it supplies is also called electromotive force, or emf, symbolized by V_{emf}. In circuit diagrams, a battery is represented by the following symbol:



- An ideal battery is just a voltage source; however, a real battery acts like an ideal battery in series with a small resistance, which is called the internal resistance of the battery.
- The internal resistance of a battery is usually $< 1\ \Omega$. However, for a "dead" battery the internal resistance goes way up.

Resistor

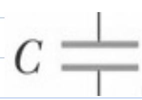
- Resistors are just circuit elements that have resistance. They are indicated in circuit diagrams by the following symbol:



- Resistors obey Ohm's Law, so the voltage across a resistor is always equal to the instantaneous current through it multiplied by its resistance. Recall that if no current flows through a resistor, there can be no potential drop across it.
- Resistors connected in series add: $R_{eq} = R_1 + R_2 + \ldots$
- Resistors connected in parallel add inversely: $1/R_{eq} = 1/R_1 + 1/R_2 + \ldots$
- The power delivered to a resistor can be calculated by $P = i\Delta V = i^2R = \Delta V^2/R$ (all equivalent due to Ohm's law). This is the rate at which electrical energy is being converted to heat energy.

Capacitors

- Capacitors are circuit elements that store charge, consisting of two separated conductors (usually taken to be adjacent parallel plates). They are represented by the following symbol:



Wire

- A wire is the simplest possible circuit element. It is just a conductor which connects two points. A wire is represented by a straight line on a circuit diagram.
- A wire can be thought of as a resistor with a resistance of $0\ \Omega$ (in practice, they have a tiny but non-zero resistance). From Ohm's law, the voltage across it must be zero no matter how much current is passing through it. This yields the useful rule that **any two points in a circuit which are joined by a wire must be at the same potential**.

Voltmeter

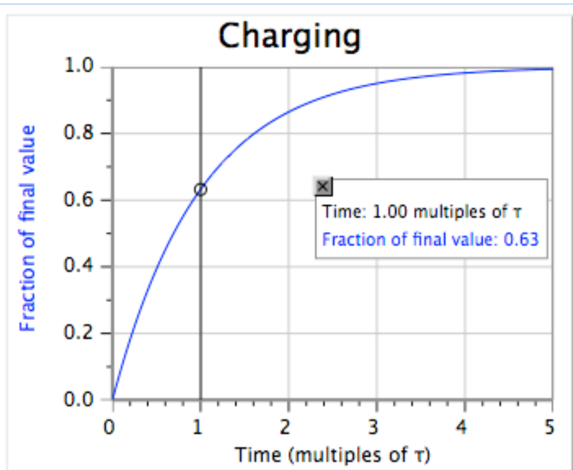
- A voltmeter is a device used to measure the potential difference between two points in a circuit. Most commonly, it measures the voltage *across* a single circuit element, such as a resistor or a battery.
- To measure a voltage, connect the voltmeter in *parallel* across the device you are interested in knowing the voltage across. (Remember, parallel devices share the same voltage difference.) Putting the voltmeter in series with your circuit will cause the circuit itself to be drastically altered and will give you meaningless results.
- A voltmeter basically acts like a very large resistor, usually on the order of megaohms. The larger the resistance, the less the voltmeter affects the circuit because it is in parallel and parallel resistances add in *reciprocal*.
- The Vernier differential voltage probe acts like a $10\ M\Omega$ resistor when placed in parallel with a circuit.

RC circuits

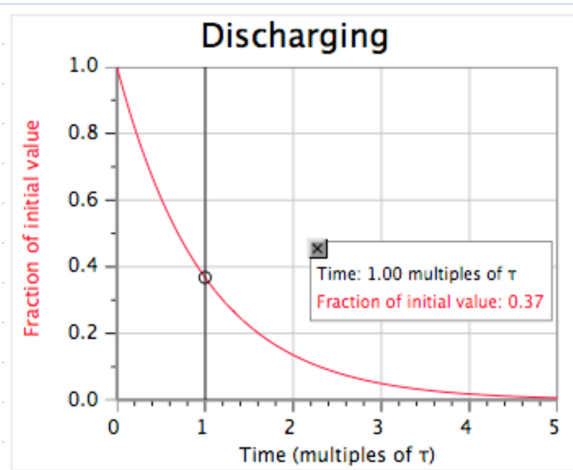
- Circuits containing a resistor and capacitor are called RC circuits. They are discussed in lecture and in section 26.5 of the text.
- RC circuits are time-dependent—that is, they are not static circuits. They asymptotically approach a steady-state limit in which all voltages remain constant and all currents go to zero. (Remember, a non-zero capacitor current means that the charge—and therefore the voltage—on the capacitor is *changing*.)
- Any voltage in an RC circuit relaxes towards its final steady-state value exponentially with time; that is, the difference between $\Delta V(t)$ and its final value decreases as $\exp(-t/\tau)$, where τ (the Greek letter tau) is called the *time constant*. In a circuit with capacitance C and resistance R, the numerical value of τ is equal to R times C. If R is in ohms and C in farads, then the product RC has units of seconds. ($1\ \Omega = 1$ V/A; 1 F = 1 C/V; so $1\ \Omega \cdot \mathrm{F} = 1$ C/A = 1 s.)
- The meaning of τ is that it is the time required for the exponential factor to become 0.37, or 37% of its original value. (Plugging in $t=\tau$ gives $\exp(-1)$, which is just $1/e$, or about 0.37.) For a voltage which is decaying to zero, τ is the time it takes for it to reach 37% of its initial value. For a voltage which is going from zero to some non-zero final value, τ is the time it takes to reach 63% of that final value (since the *difference* between ΔV and the final value decays to 37%).
- You can usually determine all of the "initial" (right after the circuit is completed) voltages by applying Kirchhoff's loop rule and remembering that *the charge on a capacitor cannot change instantaneously*. Thus the voltage across a capacitor also cannot change instantaneously; right after any switch is thrown, the capacitor must have the same voltage it had right before.
- The fact that all currents go to zero in the final steady state makes it easy to determine the final values of all of the voltages. Resistors cannot have a voltage across them without a current, so the potential difference across a resistor must be zero in this case.
- Once you know the initial and final values for some quantity X in an RC circuit (X can be a voltage, current, charge, whatever), then you can apply the exponential relaxation:

$$X(t) = X_i + (X_f - X_i)\exp(-t/\tau)$$

This equation expresses the idea that $X(t)$ starts at X_i at $t=0$ and ends up at X_f for large t. Also, either the initial or final value is almost always zero (*e.g.*, when charging a capacitor, it starts at zero; when discharging, it ends at zero), which simplifies the equation quite a bit:



$$X_{charging}(t) = X_f\,(1-\exp(-t/\tau))$$



$$X_{discharging}(t) = X_i\,\exp(-t/\tau)$$

Materials

LabPro interface with differential voltage probe

- The differential voltage probe is basically just a voltmeter that interfaces with Logger Pro. Like the multimeter you used in Lab1, it measures the potential at the red probe minus the potential at the black probe.
- A note about uncertainty: chances are good that when you connect the voltmeter to make a measurement, the digital reading will jump around a bit (especially in the last decimal place) rather than staying perfectly constant. If this happens, you can estimate the uncertainly in the voltage reading by taking half the difference of the highest and lowest numbers that it jumps between. For example, if it is fluctuating between 1.248 V and 1.262 V, you would report the voltage as 1.255 ± 0.007 V. If the reading does happen to stay constant, you can estimate the uncertainty as 1 in the last decimal place (so if it reads a constant 1.260 V, the uncertainty is ±0.001 V).

Selection of alligator clip leads

Battery pack

- This is a pack containing two 1.5-volt batteries connected in series. It can be thought of as a single 3-volt battery.
- The red lead connects to the positive terminal of the battery; the black lead connects to its negative terminal.

Selection of resistors

- The resistance of a resistor is indicated by the set of four colored stripes on the resistor. The first two bands taken together indicate the first two digits of the resistance, the third band is the multiplier (power of ten), and the fourth band is the tolerance.

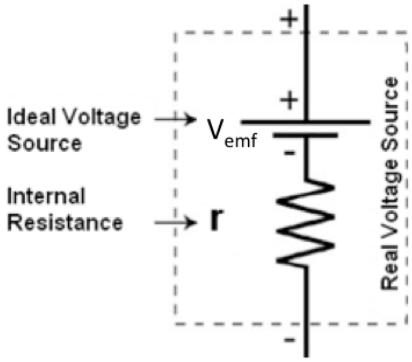
Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

- For example, the following resistor has a resistance of 26 (red-blue) times 10^5 (green), within a tolerance of 5% (gold). So its resistance is 2.6 MΩ, give or take 5%.

- If you are not sure which end to start from, most resistors have a gold or silver band as their fourth band to indicate tolerance.

- **1 "mystery" resistor**
- **1 two-terminal "ER" black box**
  - This is a box containing a battery (emf) and a small resistor, connected in series. The two exposed metal contacts A and B are the + and - terminals, respectively, of the diagram shown below:



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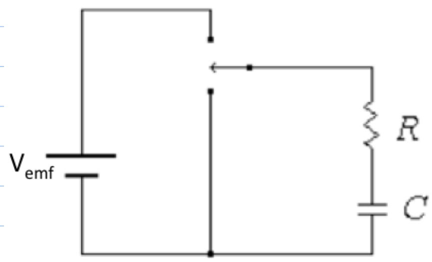
- **RC circuit board**
  - This is a board wired with the following circuit:



    where: R = 30 kΩ, C = 1 μF, and the battery is a standard 1.5-volt AA cell battery.
  - When the switch is in the up position, the battery is included in the circuit to charge the capacitor. When the switch is in the down position, the capacitor discharges through the resistor. When the switch is in the middle position, no current flows and any charge which happens to already be on the capacitor remains there.

Procedures

Charging and discharging a capacitor

- Open the Logger Pro file Lab2RCCircuit.cmbl. The file has been set up for you so that you should be ready to collect data. When you click on the  Collect button, it will collect data for 10 seconds and then automatically stop. Do not press the stop button while it is collecting.
- Familiarize yourselves with the RC circuit board. Make sure you know which switch position does what (and which component is the resistor and which is the capacitor!). It may be helpful to trace the circuit with your finger. Note that if you charge the capacitor *it remains charged*, even after you disconnect the battery, until you allow it to discharge.
- You might notice a weird behavior when collecting data of the charged capacitor: when the capacitor is charged and the switch is in the neutral (neither charging nor discharging) position, the capacitor voltage seems to decrease over time very slowly. This is because of the voltmeter itself acts like a 10 MΩ resistor connected across the capacitor. Even when the rest of the circuit is disconnected, the capacitor can discharge through this large resistor.
- To minimize any error caused by this slow discharge, keep the switch in the charging position until you are ready to throw it into the discharging position. It's okay if the voltage goes down a little bit before the discharging begins; the only difference is that it is as if the initial charge on the capacitor were somewhat lower.
- With the capacitor fully charged, press collect and then when it says "Waiting for data," throw the switch to the discharging position.

Lab 2: Report

Warm-up (10-15 minutes)

- Do the warm-up!

Who are you? (Picture and names, please)

- A:

Mystery resistor

- In this part of lab, you will attempt to determine the value of an unknown resistance by designing one or more circuits involving the mystery resistor and making appropriate measurements. Be forewarned: it's not trivial.

- The tools you will be able to use to measure the mystery resistor are:
  - A battery pack
  - A selection of resistors of different (known) values
  - LabPro interface with differential voltage probe and the Logger Pro software. (Note that the only measurements you can make are *voltage* measurements. You cannot make direct measurements of current or resistance!)

- Play around with the components at your disposal and see what you can measure. At some point, however, work with your lab partners to devise a systematic plan to determine the mystery resistance. Then put your plan into action!

- **(1) Describe your procedure here; be specific! Include snapshots of circuit diagrams if you need to. Feel free to add as many steps as you need.**
  - A:

- **(2) Record your measurements and calculations here. Include uncertainty measurements, since we will want to know the uncertainty of your final answer.**
  - A:

- **(3) What is the mystery resistance R? What is the uncertainty in your measurement of R?**
  - **R =**
  - **uncertainty in R =**

What's in this box?

- In this part of the lab, you will determine the value of the the emf, V_{emf}, and the internal resistance, r, of a battery inside a black box . The black box, labeled "ER," contains a "battery" which consists of a voltage source and a small-but-not-tiny "internal resistance." The battery is oriented so that terminal A is at a higher potential than terminal B. As with the previous section, you will only be able to measure voltage, not current nor resistance!

- **(1) Describe your procedure and record your measurements here. Include uncertainty measurements, since we will want to know the uncertainty of your final answer. Feel free to add as many steps as you need:**
  - A:

- **(2) Record your results along with a copy of any graphs you used here. Don't forget uncertainties!**
  - **V_{emf} =**
  - **r =**

Charging and discharging of a capacitor

In this part of the lab you will use an RC circuit board to explore the processes of charging and discharging a capacitor. You will also find the time constant for this circuit. Make sure you familiarize yourself with the RC circuit and the LoggerPro file as described in the "Materials" and "Procedures" parts of the write up before taking data.

Charging RC

- Start with the capacitor fully discharged. Connect the differential voltage probe in order to measure the voltage across the capacitor and click on Collect. When you see the message "Waiting for data," throw the switch into the charging position.

(1) What are the "initial" and "final" values of the capacitor voltage?

- A:

(2) What functional form does the plot look like?

- A:

(3) How long after you throw the switch does it take for the capacitor voltage to go 63% of the way from its initial to final values?

- A:

(4) Is this consistent with the stated component values for R and C? Explain.

- A:

(5) Paste the relevant part of your graph here. Be sure to display the results of your fit:

- A:

Discharging RC

- Start with the capacitor fully charged. To minimize any error caused by slow discharge (see "Procedures" and Warm-up for explanation), keep the switch in the charging position until you are ready to throw it into the discharging position. It's okay if the voltage goes down a little bit before the discharging begins; it is as if the initial charge on the capacitor were somewhat lower. Click on Collect and when you see the message "Waiting for data", throw the switch into the discharging position.

(1) How long does it take for the capacitor voltage to become 1/e of its initial value?

- A:

(2) Is the time constant the same as it was for charging? Explain.

- A:

(3) Paste the relevant part of your graph here. Be sure to display the results of your fit:

- A:

Conclusion

What is the most important thing you learned in lab today?

- A:

- Don't forget to save your file as a pdf and upload it to the isites dropbox!
When printing this file, make sure you only print **page 4**!