Lab 5: Harmonic Oscillations and Damping

I. Before you come to lab...

II. Background

III. Introduction

- **A.** In this lab, you'll get a chance to explore the oscillations of a mass-spring system, with and without damping. You'll get to see how changing various parameters like the spring constant, the mass, or the amplitude affects the oscillation of the system. You'll also see what the effects of damping are, and explore the three regimes of underdamped, critically damped, and overdamped systems.
- **B.** Objectives for this lab:
  - **1.** Learn how to numerically characterize oscillatory motion
  - **2.** See how springs work in combination, both in series and in parallel
  - **3.** See how the oscillation frequency depends on k and m
  - **4.** Add damping to a harmonic oscillator system and observe its change in behavior
  - **5.** Vary the amount of damping to see the three different damping regimes

IV. Materials

- **A.** 2 long, flimsy springs (type "A")
- **B.** 2 short, stiff springs (type "B")
- **C.** 1 type A spring with a "collar" on one end
- **D.** Mass stand and masses
  - **1.** The mass stand is a hook with a tray at the bottom for putting masses on it. The stand itself has a mass of 50 grams.
  - **2.** There are also masses ranging from 100 g to 500 g in the set.
- **E.** Lab jack
- **F.** Sonar motion detector
  - **1.** The sonar motion detector is a sensor that detects the position of objects using sonar ranging:

  - **2.** The **minimum distance** away from the sonar detector that objects can be "seen" is 15 cm (about 6 inches). The resolution of the detector is 0.3 mm (that is, an object has to move by at least 0.3 mm in order for the sonar detector to read a different position measurement for it).
  - **3.** The sonar detector connects to the LabPro interface and works with Logger Pro.

4. The detector comes with a mounting unit that can be used to clamp it to something, or it can be unscrewed from the mounting unit entirely.

- **G.** 600 mL beaker
- **H.** 25 mL graduated cylinder
- **I.** Plastic water bottle
- **J.** 16 oz bottle of karo corn syrup
- **K.** Stirring rod
- **L.** Forceps

V. Procedure

A. Before you begin:

1. Take a picture of yourselves using Photo Booth and drag it into the space below:

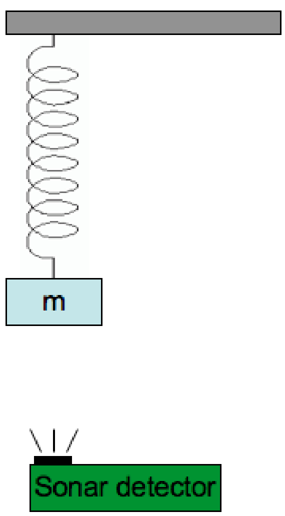
2. Tell us your names:

B. Measuring harmonic motion

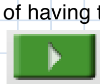
1. Open the file Lab5.cmbl in Logger Pro.

2. One spring (type A)

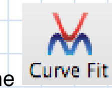
a. Hang a type A (long, floppy) spring from the overhead beam and use it to suspend a total mass of 150 g (50 g for the stand + 100 g extra) and set it in oscillatory motion up and down, with an amplitude of a few cm. Place the motion detector on the table below the oscillating mass, facing up. (Be careful, try not to drop anything onto the detector.) The setup should look approximately like this:



b. Remember, the sonar detector has a **minimum distance of 15 cm** before it can detect objects. Do not place it too close to your oscillating mass. Also, use only small amplitudes of oscillation; this will significantly decrease the chances of having the mass fall off the stand and break something.

c. In Logger Pro, click on  Collect and observe the mass's motion for a few periods. You should see a nice sinusoidal motion. If you don't see anything, you may need to re-scale the y-axis on the graph. If you see something more complicated than a simple sine wave, you probably have an excessive amount of side-to-side motion in your oscillation. Try to get everything line up nice and vertical.

d.

Try fitting a sine wave to the position vs time graph. Use the 

button and under "General equation," scroll down to the option near the bottom called "Undamped." (If you can't find this option, you are using the wrong Logger Pro file.) What is the angular frequency of the oscillation? (Hint: the general equation for undamped motion is position = A*sin(C*t+D)+E. Which of the parameters corresponds to angular frequency?) Don't forget to include the correct units.

angular frequency (ω) =

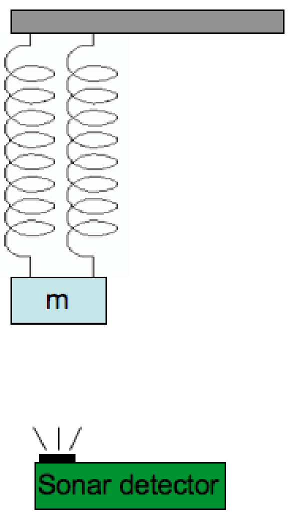
e. Paste a copy of the position vs time graph, including the sinusoidal fit, below:

f. From the value of ω, calculate the spring constant of the type A spring (don't forget units):

k_A =

3. Two A springs in parallel

a. Now connect two type A springs in parallel (side-by-side) to suspend the same 150 g mass, like so:



(You'll need to put one end of each spring through the loop at the top, and one end of each through the hook on the mass stand. It's okay if the two springs are touching.)

b. Repeat the experiment, and calculate ω and $k_{effective}$.

(1) ω =

(2) $k_{effective}$ =

c. Compare this result to k_A. Does this agree with what you found on the pre-lab?

4. One B spring

a. Now do the same thing for a single type B spring. You'll need to use a larger mass because the B spring is much harder to stretch; try a total mass of 550 g.

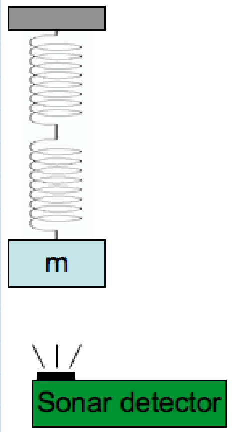
b. Measure ω and calculate k for the type B spring:

(1) ω =

(2) k_B =

5. Two B springs in series

a. Place two B springs in series (end-to-end) as shown:



b. Repeat the experiment, and calculate ω and $k_{effective}$.

(1) ω =

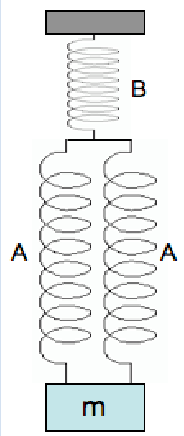
(2) $k_{effective}$ =

c. Compare this result to k_B. Does this agree with what you found on the pre-lab?

d. Does it seem like the two springs stretch by the same amount as the mass oscillates?

6. Three springs, in series/parallel

a. Connect one B and two A springs in the configuration shown below:



b. Before you actually measure anything, predict what value of $k_{effective}$ you would get from this combination: $k_{effective}$ (predicted) =

c. Now actually perform the experiment, using a mass of 250 g.

(1) ω =

(2) $k_{effective}$ =

(3) How well does this agree with your prediction?

d. Compare the amount that spring B stretches with the amount that the A springs stretch. Are they equal? If not, which amount is larger?

7. Something to think about

a. Suppose you took a type A spring and cut it in half, to make two shorter springs. What would the spring constant of one of these springs be?

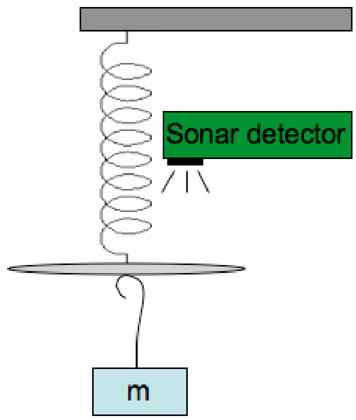
$k_{A \text{ cut in half}}$ =

b. If you are curious, we have some half-A springs lying around for you to test your prediction.

C. Exploring damping

1. Damping due to air drag

a. Now connect the spring with a "collar" to the overhead beam and set up the sonar detector on the mounting unit **above** the collar and facing down, as shown here:



b.

Use m = 150 g and set the oscillation going with a period of a few centimeters. Click  Collect and observe the motion of the mass over the course of 15 or 20 seconds.

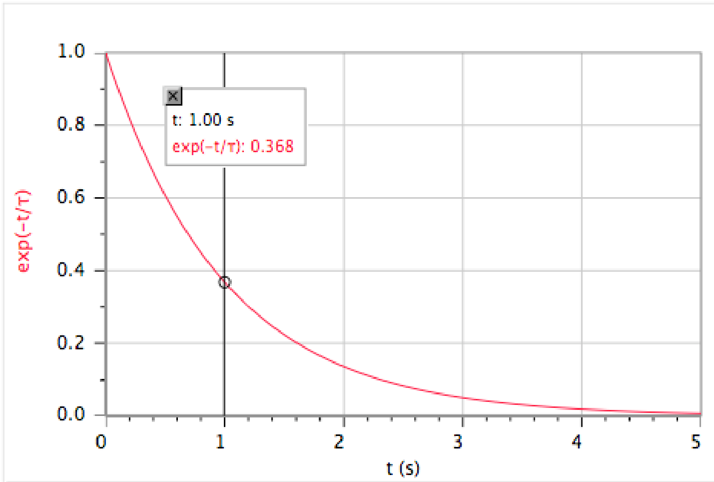
c. You should notice that the mass still oscillates sinusoidally, but the amplitude gradually decreases over time. This is because the collar has a significant amount of air drag, which provides a moderate amount of *damping*. Try fitting the curve called "Underdamped" to the position vs time graph.

(1) This general function is a sine wave whose amplitude decreases exponentially with time: position = A*exp(-t/B)*sin(C*t+D)+E.

(2) The key difference between this and undamped motion is the exponential factor, exp(-t/B). The parameter B is called the *time constant* of the motion; it represents how quickly the amplitude decreases. The time constant is usually denoted with the Greek letter tau (τ), but Logger Pro doesn't do Greek letters in curve fitting, so that's why it's B in the equation.

(3) In general, the time constant for damped motion depends on the amount of damping: for underdamped motion, increasing the amount of damping will cause the amplitude to decrease faster.

(4) Quantitatively speaking, τ is equal to the amount of time it takes for the amplitude to decrease by a factor of 1/*e*, where *e* is the constant 2.718, the base of the natural logarithm. (1/*e* is about 0.37.) Here's a graph of $\exp(-t/\tau)$ for τ=1.00 second:



d. Paste a copy of your position vs time graph, including the fit parameters, here:

e. What is the angular frequency of the oscillatory motion?

ω =

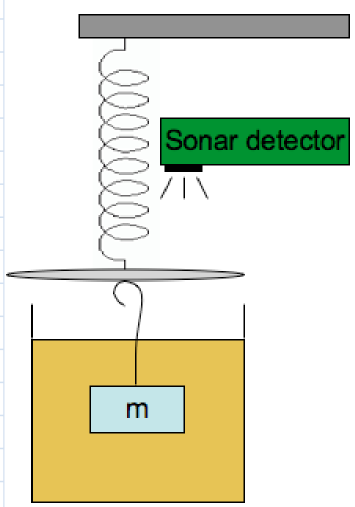
f. What is the time constant for the motion?

τ =

g. Looking at the graph, compare the amplitude of the motion at the beginning, and at a time B seconds later. Do they differ by a factor of about 2.7?

2. Damping due to viscous drag

a. Fill the 600 mL beaker with karo syrup (use the whole bottle) and set it on the lab jack. Keep the spring with the collar on it, and the 150 g mass. Position the mass over the beaker and raise the height of the jack until the mass is resting in the center of the beaker as shown:



b. Motion of heavily damped systems

(1) With the mass motionless at equilibrium, try to position the collar as level as possible and then zero the motion detector by clicking . It will click a few times and then be still.

(2)

Start the data collection by clicking Collect. Without touching the collar directly, pull the mass up so that the top of the mass is level with the top of the syrup and then release it. You should see it gradually return to its equilibrium position, but it will take quite some time. After it has come to a complete stop, you can click on the Stop button to stop the data collection.

(3) Note that this motion is totally different from what you have seen before. There is no oscillation of any kind; instead, the mass just gradually relaxes back in to its initial position. This is a characteristic of *overdamped* motion: the damping is so great that not only is any oscillation killed out, but it also takes a long time to return to equilibrium.

(4) Using the mouse, click and drag in the graph to select the time interval from just after the moment you released the mass until the end of the data collection. Click on Curve Fit and choose the general equation called "Overdamped." Check the box marked "Time offset." This will effectively use the moment of release (the beginning of your selected time interval) as t=0, instead of the time you clicked Collect. Click the button for Try Fit. You may have to wait for a few seconds while the program calculates the best fit. If the function looks like it closely fits the data, click on OK; if not, you may need to click Try Fit again. Ask a TF for help if you are having trouble getting a good fit; you may need to adjust the endpoints of your fitting interval.

(5) When you have a good fit, record the best-fit value of τ (the time constant) here:

τ =

(6) Double-click on the graph and under Graph Options, check the box for "Y Error Bars." Click OK. You should now see tiny error bars take the place of each data point. (You may need to zoom in on the vertical scale in order to see them.) These error bars represent the sensitivity of the motion detector.

(7) From the graph, estimate when the mass reaches a position within one error bar of its final equilibrium position. Subtract the starting time (t when you released the mass) to get the time elapsed until the mass reaches equilibrium:

Time to equilibrium =

(8) Turn to page 2 of the Logger Pro file. In the data table there, enter your time constant in the top row, in the column labeled τ. Enter your time to equilibrium in the last column, labeled "Time to zero." (The third column, ω, doesn't have a defined value because overdamped systems do not oscillate, and hence have no angular frequency.)

(9) Turn back to page 1 in the Logger Pro file. Press Apple-L to store this data set. It is now called "Run 1."

c. Now remove the mass from the beaker and take the beaker off the lab jack. **Don't** change the height of the lab jack. Add 20 mL of water to the karo syrup and stir it until it is well-mixed. Because viscous fluids do not mix easily, this will take about 60 seconds of stirring. The effect of adding the water will be to reduce the viscosity and therefore lower the amount of damping in the system.

d. When the water is thoroughly mixed in with the syrup, put the beaker back on the lab jack and the mass back into the beaker. Repeat step b (starting with zeroing the motion detector) for this mixture. Record the time constant and time to zero in the table on page 2 of the Logger Pro file, this time in the row marked "20 mL dilution." Be sure to remember to store each data set (Run 1, Run 2, Run 3, etc.) as you go. If the graph gets too crowded for you, you can double-click on it and uncheck the boxes for the runs you don't want to show.

e. Keep diluting by 20 mL at a time, mixing thoroughly, and observing the motion, recording your measurements on page 2 of the Logger Pro file. At some point, you will notice that when you release the mass, instead of just settling back to equilibrium, it actually **overshoots** equilibrium slightly and then comes back. When you see this happen, you'll know you have passed into the region of *underdamped* motion. (If you're not sure if what you're seeing constitutes underdamping, ask your TF.)

For underdamped motion, use a different general equation for the curve fit. Unsurprisingly, you should use the one called "Underdamped." (You should continue to use the Time Offset option.) This motion is qualitatively similar to what you saw when you let the mass with the collar oscillate in air, except there are many fewer oscillations before the motion dies out. There are two important changes in the procedure when you reach underdamping:

- **(1)** In calculating the time to zero, don't measure to the *first* time the motion reaches equilibrium. Instead, measure to the time when it reaches equilibrium *and stays there*.
- **(2)** When you fit the equation for underdamped motion, you can also determine the angular frequency of the oscillation, ω. Enter a value for ω into the data table on page 2 of the Logger Pro file for every dilution which falls into the underdamped regime.

f. After you have completed all of the dilutions up to 100 mL, go to your data on page 2 and see which one came closest to **critical damping** (the threshold between over- and underdamped motion).

- **(1)** Which dilution was it?
- **(2)** What was the time constant for critical damping? the time to equilibrium?

  τ =

  Time to zero =
- **(3)** Paste a graph of position vs time for your critically damped system here:
- **(4)** Create a graph showing both τ and Time to zero vs dilution and paste it here:

  In words, what conclusions can you draw from this graph?
- **(5)** Look at the values of ω that you observed for the underdamped systems. How do they compare to the ω you got for the same system damped only by air drag? For underdamped systems, does ω increase, decrease, or stay the same as the amount of damping goes up?

VI. Conclusion

- **A.** When you have finished, clean up anything in your work area that has has karo syrup all over it (the beaker, mass stand, masses, stirring rod, forceps if you used them, and anything else which was in the splash radius of your karo syrup) by taking it to the sink and rinsing it out thoroughly with warm water. Leave the glassware to dry on the tray **at your station** (not at the sink).
- **B.** Submit your lab report online according to the instructions on the plastic sheet at your computer. When you get to the step where you export to HTML, put it in the folder within Sites called **Lab5**. Make sure to select "Current Page" instead of "All," and leave the box for cover page **unchecked** before clicking Choose.
- **C. Super-duper important—don't even think about skipping this step!** Before you leave the lab, every member of your lab group should open a browser and go to

  http://physci.fas.harvard.edu/~yourFASusername

  and make sure that your lab report is there under the link called "Lab 5." If not, then you haven't submitted it correctly; ask a TF for help. If your lab report isn't submitted, you won't get credit for doing the lab.
- **D.** Maybe this week, you could gather up all of your things and take them with you when you leave the lab. You know, calculators, notebooks, cell phones, iPods, lab handouts, even pencils & pens. It's just a thought.