

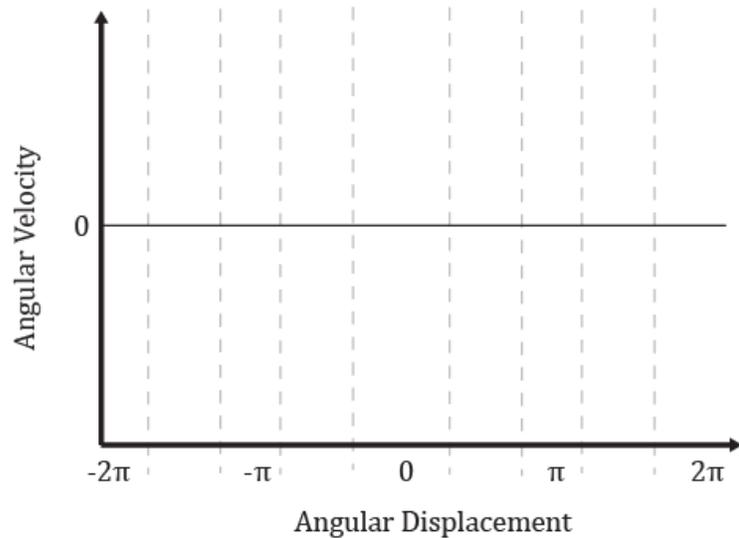
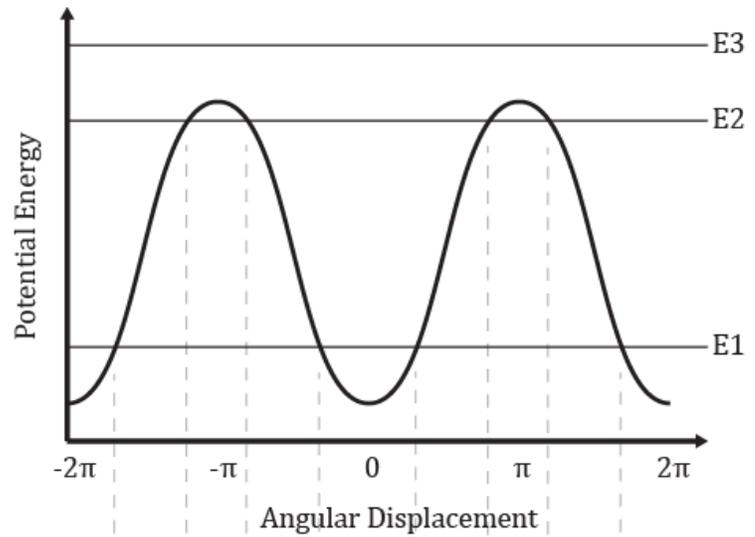
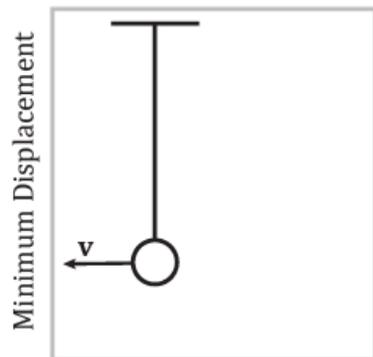
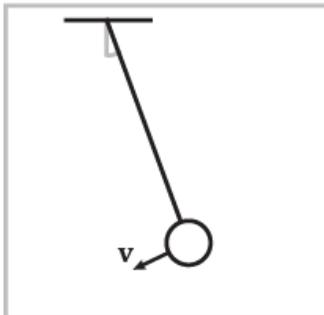
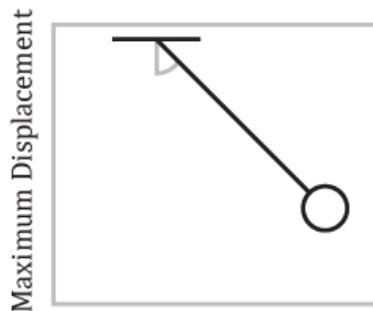
Lab 10: Phase Space and the Pendulum

In this lab we will be investigating the phase space and nonlinear behavior of the pendulum system.

Part 1: Concept Review

Q1: Draw the total acceleration vector on the pendulum mass for each snapshot in time

Q2: Draw contours of the pendulum phase space diagram for energies E_1 , E_2 , and E_3 labeled on the potential energy graph.



Part 2: MATLAB Phase Space Plotter

Download `phase_space_plotter.m` from the lab materials page on the course site. Look over the script and then try to run it. Click around on the plot and see what happens!

Q3: The position coordinate x is measuring an angular displacement, so it should be periodic in 2π . Change the range of the x-axis so that it runs from $-\pi$ to π , and add code after line 27 to force x to be periodic in 2π . Hint: you might find the `mod` function useful. Write down your added code below:

Q4: Look at the phase space curves generated by the following four points. Describe what you would see the pendulum doing:

$$(x=0, v = 4)$$

$$(x=0, v = -4)$$

$$(x=0, v = 8)$$

$$(x=0, v = -8)$$

Part 3: Dimensional Analysis

Dimensional analysis is an extremely useful tool in physics. You can use dimensional analysis to intelligently guess the answer to a problem, and it will almost always be correct (up to some dimensionless coefficient). For example, a spring system has parameters k and m which have units:

$$[k] = M/T^2 \quad [m] = M$$

If we wanted to find the angular frequency of the system (which has units $[\omega] = 1/T$) without solving any pesky equations we can just look at k and m and try to combine them in a way that gives us the units of ω . So:

$$[k]/[m] = (M/T^2)/(M) = 1/T^2 = [\omega]^2$$

$$\text{And thus } \omega = \sqrt{k/m}$$

Q5: The pendulum system has the parameters:

$$[g] = L/T^2 \quad [m] = M \quad [L] = L$$

Unlike the spring system, the displacement for a pendulum is measured in radians and thus is essentially dimensionless. You will need to tack on a dimensionless parameter $f(\theta)$ to the answer that you get through dimensional analysis. Write ω in terms of g , m , L , and $f(\theta)$ (show your work!):

$$\omega = \underline{\hspace{10em}}$$

Part 4: Nonlinear Behavior

With the pendulum setup and Logger Pro you can measure angular amplitude and frequency of the pendulum.

Q6: How will you determine $f(\theta)$?

Q7: Make a plot of $f(\theta)$ vs θ . Print and attach or upload to the course dropbox.

Q8: What is $f(\theta)$ for small values of θ ?