

Lab 4 and 5 Summaries

Physics 12a

Lab 4: χ^2 and Fitting

In Lab 4 we fit three different models to data based on three different air drag models. A model for $x(t)$ with no drag can be found by solving the differential equation:

$$m \frac{d^2x}{dt^2} = -mg$$

A model with linear drag can be found by solving the differential equation:

$$m \frac{d^2x}{dt^2} = -mg - \alpha \frac{dx}{dt}$$

And quadratic drag follows from:

$$m \frac{d^2x}{dt^2} = -mg - \alpha \left(\frac{dx}{dt} \right)^2$$

Note that these equations will vary depending on your choice of coordinate system. In the above examples the force of gravity points in the negative x direction. The drag force should always point opposite the velocity vector (which can be tricky to incorporate in the quadratic drag case).

By fitting these different models to data (using MATLAB's `cftool`) we can evaluate which fit is the best by looking at the residuals and the value of the reduced chi-squared χ_R^2 :

$$\chi_R^2 = \frac{1}{N - n} \sum_i \frac{(x_i - m_i)^2}{\text{sigma}_i^2}$$

Where N is the number of data points, n is the number of fit parameters, the x_i 's are the measured values, the M_i 's are the model values, and the σ_i 's are the uncertainty on the measured values. Some examples of how to interpret χ_r^2 for different fits and data are given in Figure 1:

- Diagram A depicts a model with terribly systematic residuals and large χ_R^2 , indicating that the model is a poor match for the data.
- Diagram B depicts a model with random residuals and large χ_R^2 , indicating that the model is a poor match for the data or uncertainty has been grossly underestimated.

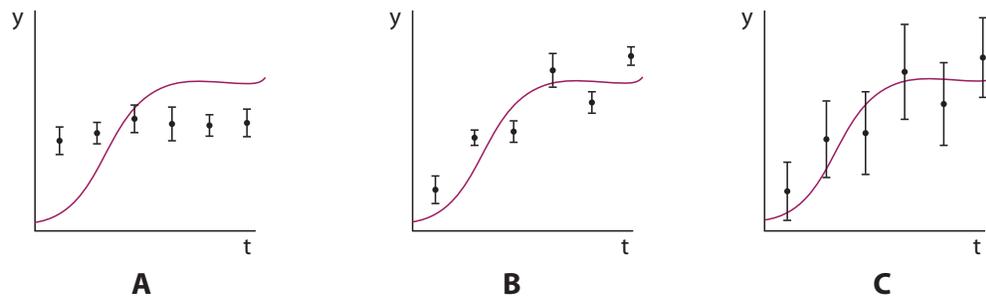


Figure 1: Model vs. data.

- Diagram C depicts a model with random residuals and small χ_R^2 , indicating that the model is over-fitting the data or uncertainty has been overestimated

In general, for a good fit $\chi_R^2 \sim 1$.

With good residuals and reasonable χ_r^2 one can be confident that the model does a good job describing the dynamics of the system. In this case the fit parameters measure aspects of the system indirectly. for example, if the quadratic drag model fits the data very well then one can extract the drag coefficient α or the terminal velocity v_T of the ball without directly measuring either of these properties.

Often a theoretical model must be adjusted to account for imperfections in data-taking devices. The lack of a timed release system in this lab meant that students needed to trim the data, resulting in the inclusion of extra fit parameters (see Figure 2 caption for detailed explanation).

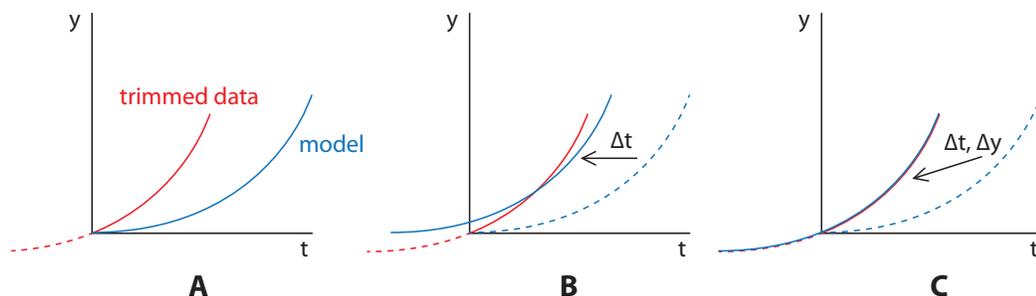


Figure 2: Adding additional fit parameters to compensate for data trimming. Due to trimming many students needed to introduce an additional parameter to account for the mismatched origins for their data vs the model. Only shifting in t results in a nonzero $y(1)$, so they needed to introduce an additional fit parameter to shift y .

Lab 5: Center of Mass, Energy, and Changing Frames

The center of mass COM of a system is a weighted average of objects with positions x_i and masses M_i in the system:

$$COM = \frac{\sum_i x_i M_i}{\sum_i M_i}$$

When a system experiences no net external force the COM velocity will be constant, even if the forces between the objects of the system are very complicated (for example, connected by a spring). This is why we can ignore intermolecular forces when we want to predict the motion of a macroscopic object.

Momentum is always conserved between collisions of moving objects, but kinetic energy is only conserved when the collision is elastic. Conversely, kinetic energy not conserved when the collision is inelastic. Instead, the kinetic energy goes into deforming the objects, creating sound waves, and generating heat.

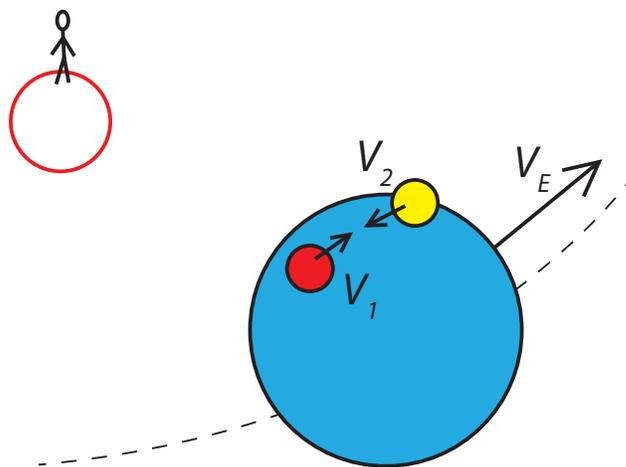


Figure 3: In the Earth's rest frame, the Earth moves at zero velocity and the red and yellow balls roll towards each other with velocities V_1 and V_2 , respectively. However, someone on the Sun would see the Earth moving with velocity V_E , and would see the red ball moving with velocity $V_E + V_1$ and the yellow ball moving with velocity $V_E - V_2$. Shifting between these two reference frames is called a Galilean transformation, and involves the simple addition and subtraction of velocities