Lab 1: Measurement and Uncertainty

▽**I. Before you come to lab...**
  ▽**A.** Read through the handout on the course website, Chapters 1-2 from Taylor, *An Introduction to Error Analysis*. These chapters will introduce the main ideas behind measurement and uncertainty that we will explore in this lab and throughout the semester. Although only the first two chapters are required reading, the full text is on reserve at Cabot and contains detailed explanations of the main concepts from experimental science. Other references you may find helpful:
   1. B. S. Massey, *Measures in Science and Engineering*
   2. S. K. Kimothi, *The Uncertainty of Measurements*
  B. Read through this handout in its entirety.
  ▽**C.** Download the Logger Pro software and learn how to use it.
   1. Logger Pro for Mac or Windows can be obtained from
     http://downloads.fas.harvard.edu/download
     You will need to log in with your Harvard ID and PIN. Logger Pro 3.8.2 requires Mac OS X version 10.4.11 or 10.5.1 or later, or Windows XP/Vista/7. If you have a PC or Mac but are unable to install Logger Pro, contact Joon for help. His email address is pahk@fas.harvard.edu. If you don't have a PC or a Mac of your own, you can use Logger Pro on a computer in the Science Center computer lab to complete this part.
   2. To familiarize yourself with Logger Pro, install the software and then work through some of the tutorials.

     You can do this by starting Logger Pro and then clicking on the [Open] in the toolbar. The Open File dialog box will open to a folder called "Experiments." Inside that folder, find the sub-folder called "Tutorials." Here are the tutorials that you should do:
     1 Getting Started
     5 Manual Data Entry
     7 Viewing Graphs
     8 Stats, Tangent, Integral
     9 Curve fitting
  D. Complete the pre-lab question as part of Homework #1, and turn it in with the rest of your homework by 9:30 am Tuesday.
  ▽**E. On the day of the lab...**
   1. Come on time to Science Center 115.
   2. Bring this handout.
   3. Bring a writing implement.

▽**II. Introduction**
  A. This first lab is intended as a gentle introduction to the ideas of measurement and uncertainty. You'll measure the periods of 6 pendulums using a stopwatch and see how the uncertainty in the measurement depends on the number of times you repeat the measurement.
  ▽**B.** Learning objectives for this lab:
   1. Understand intuitively what uncertainty is and what determines the uncertainty of a measurement
   2. Understand what is learned by repeating a measurement, and be able to figure out how many times you should repeat it
   3. Understand that a measurement by itself, even with uncertainty, is meaningless unless compared to a

prediction or to another measurement

III. Background

(If you haven't already read the chapters from Taylor posted on the course website, do so now before continuing.) In an experimental setting it is just as important to specify not only what you know, but how well you know it. The purpose of this first lab is to introduce some of the basic ideas behind the study of uncertainty.
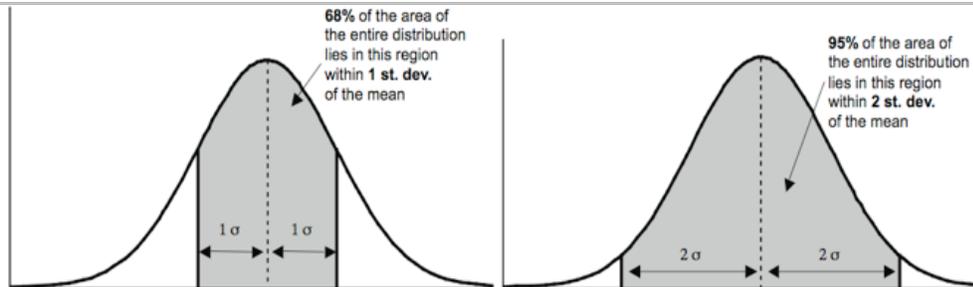
A. Gaussians

1. You know from reading the Taylor excerpt that all measurements are subject to some random variation, which we refer to as "uncertainty." It turns out (for reasons we will see later in the semester) that the vast majority of the time, this random variation is distributed according to a "bell curve" like the following:
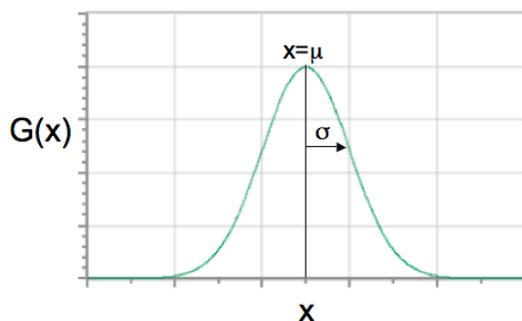
This function has lots of names, including bell curve, normal distribution, and Gaussian distribution. We'll use the name Gaussian (pronounced GOW-see-an) in this course. The Gaussian is named for K.F. Gauss, a 19th-century pioneer in math and physics.

2. We won't go into a lot of probability theory here, but the most important qualitative thing to observe about the Gaussian distribution is that it's peaked around a central value, called the **mean**. In the graph above, the value of the mean is represented by the Greek letter μ (mu). The significance of this shape is that if you repeat a measurement lots of times, most of the values will be "close" to some central value, the mean.

3. We can be a little bit more quantitative about what we mean by "close." A Gaussian distribution is characterized not only by its mean μ, but by a second parameter called the **standard deviation**, represented by the Greek letter σ (sigma). The standard deviation (which is always positive) is a measure of the width, or spread, of the distribution. The smaller σ is, the more sharply peaked the distribution. A large value of σ corresponds to a wide, flat distribution, making it harder to identify the peak.

4. Rule of 68 and 95

Using the mean and standard deviation, we can state the following two numerical rules about measurements that follow a Gaussian distribution:

a. 68% of the time, the measurement falls between μ-σ and μ+σ, i.e. within one standard deviation of the mean. In other words, it has about a 2/3 chance of being within "one sigma" of the mean.

b. 95% of the time, it falls between μ-2σ and μ+2σ. It is quite rare for a Gaussian variable to have a value which is more than 2σ from the mean. This is illustrated below:

68% of the area of the entire distribution lies in this region within **1 st. dev.** of the mean

1σ 1σ

95% of the area of the entire distribution lies in this region within **2 st. dev.** of the mean

2σ 2σ

5. Here is the most important thing to know about the connection between the Gaussian distribution and measurement uncertainty: when we state the value and uncertainty of a measurement, **the uncertainty corresponds to a 68% confidence interval**, by convention. You can think of this as a quantitative refinement of the general idea that we are "reasonably certain" that the measured value lies within the uncertainty range.

a. For example, if I state that a certain distance is 2.77 ± 0.02 cm, that means that I think repeated independent measurements of the same distance would lie between 2.75 and 2.79 cm about two-thirds of the time.

b. Given what we know about Gaussians, I could make the further statement that the distance would be measured to be between 2.73 and 2.81 cm (2σ away from the mean on either side) about 95% of the time.

c. *Warning*: even though we define the uncertainty range as the one that corresponds to a 68% confidence interval, that doesn't mean that the uncertainty is always equal to the standard deviation. "Why not?" you ask. For the answer, read section C on Standard Deviation and Standard Error.

6. In case you're curious, the Gaussian distribution function has the following mathematical form:

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

The weird-looking factor outside of the exponential is there for normalization; most of the time, we won't care about it. The important parts are the things inside the exponent. Actually, most of the time, we won't really care about the functional form of the Gaussian at all, but it is useful to be able to fit a Gaussian to a histogram to estimate the mean and standard deviation of a set of measurements.

B. Uncertainty of a single measurement: Reading error

1. During this lab you will learn how to calculate the uncertainty of a measurement by repeating the measurement many times and generating a histogram of results. Is this the only way to determine the uncertainty? Luckily, the answer is no. For many measuring devices, you can get a very good estimate of the uncertainty just from common sense.

2. The simplest example is using a ruler to measure the length of something. Here's an example: suppose you use this ruler to measure the position of the left edge of the surface shown:
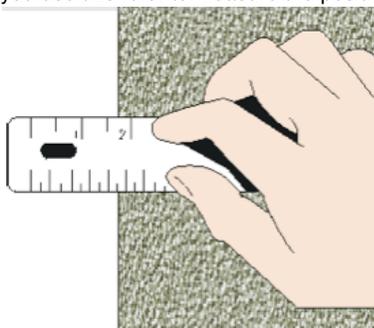
It looks like it's a bit under 1.75 inches. But how do we get the uncertainty? *We have to estimate the*

minimum and maximum values the reading could have without us noticing the difference.

   a. Here's how I would do it: the upper bound seems to be 1.75" or maybe 1.74"; the lower bound is trickier, but I'd guess it's about 1.70". (The two ruler marks on either side are 1.50" and 2.00"; it's pretty clearly more than 40% but less than 50% of the way from 1.50 to 2.00.) So I would say that the position is 1.72 ± 0.02 inches; the uncertainty of 0.02" is called the *reading error* of the measurement.

   b. Another measurer might well disagree, but hopefully not by a lot. Determination of reading error can be a little crude, but the important thing is to get a reasonable estimate of the uncertainty without having to do the measurement many times.

   3. Not all reading errors are created equal. For example, using the same ruler to measure the position of a rough edge (rather than a smooth one) would give a larger reading error. Ultimately it always comes down to your judgment and common sense.

   4. Once you have estimated the reading error on a measurement, you can treat it just like a statistical uncertainty, as if the reading error were a standard deviation of repeated measurements. That may be a stretch, but estimating reading error is a quick and dirty approach to uncertainty that produces generally reasonable results.

C. Standard Deviation and Standard Error

   1. If the data we take can be reasonably approximated by a Gaussian, then we know that about 68% of the measurements will be within 1σ of the mean, and 95% within 2σ. Therefore, when we use a *single* measurement as a guess for the mean value, the uncertainty of that guess is just σ, the standard deviation.

   2. If we take repeated measurements, the *best* guess we can make for the true value is the average (mean) of all the measurements. Intuitively, it should make sense that the more times we repeat the measurement, the better this guess becomes. The uncertainty of this best estimate is related to the **standard error** (or SE) which has the formula:

$$\text{Standard Error} = \frac{\sigma}{\sqrt{N}}$$

   The N in the formula is the number of repetitions that get averaged to produce our best guess. Essentially, **the SE represents the uncertainty of our best guess**.

   3. Note that when N=1, the SE is equal to the standard deviation. This confirms our earlier statement that the uncertainty of a single measurement is equal to the standard deviation.

   4. Standard error is useful for knowing how much data you *should* take in any given experiment. The more data points you take, the smaller the standard error will be. That being said, there are two major caveats:

   a. The formula for SE produces diminishing returns. Making a measurement ten times instead of once gives you about a factor of 3.1 improvement on your uncertainty. That may seem quite attractive in many cases. But if you want another factor of 3.1 improvement, you have to do it 100 times, and really, who wants to do the same thing 100 times in a row?

   b. You should go into an experiment with an idea of an acceptable tolerance for the uncertainty. For example, consider a machinist creating a tube for a ball bearing that needs to be 5.0 mm in diameter. If she is off by as much as 1 mm, then the ball bearing either won't fit or will rattle down the tube. But the likelihood of him making it exactly 5.0(000000...) mm on the nose is pretty small. As long as she is within a couple of hundredths of a millimeter, the part will still work correctly in the machine. This allowable deviation, or tolerance, in her measurement is analogous to knowing the allowable standard error in our answer and, therefore, how much data to take.

IV. Materials

A. Pendulum

B. Digital stopwatch

1.

2. Press the right button to start or stop the timer.

3. Press the left button to reset the timer to zero.

▽**C.** LabPro interface

1.

2. The LabPro interface is the green box that allows you to connect a wide variety of different sensors to the computer via the Logger Pro software. Today, you'll be using it with the photogate.

▽**D.** Photogate

1.

2. The photogate connects via the digital channels ("Dig/Sonic") to the LabPro interface. Logger Pro should recognize the photogate upon connection.

3. A beam of infrared light from one arm of the photogate is directed to a detector on the other arm. When this beam is blocked (e.g., by a hand or, say, a pendulum bob), a change in signal will be registered.

▽**V. Procedure**

▽**A.** Before you begin...

1. Open the Photo Booth application and take a picture of your lab group. Drag it into the space below:

2. Type in your names (from left to right in the above photo):

3. Make sure you answer all of the questions in the boxes below. (Even though they are bold-faced, it's easy to gloss over some of them.) If you are unsure of what you are being asked to do/answer, ask your TF.

▽**B.** Pendulum

In the first part of the lab, you will attempt to determine the periods of different pendulums. We are not as interested in the period itself as we are in the *uncertainty* in the determination of the period.

1. Each member of your group should get a stopwatch and a piece of paper. You will each measure the period of the pendulum several times. Before measuring anything, spend some time talking amongst your group about the best way of measuring the period. Once you have all decided upon a method, stay consistent for all of the measurements you take today. **Write a short description of your timing method here**:

2. Measuring one period, numerous times—Stopwatch

   a. Again, make sure you are all using the same technique to time the pendulum.

   b. Your TF will assign your group to one of six pendulums in the lab. **Measure the length of the pendulum, including uncertainty due to reading error.** Don't forget to include units on all of your measurements and uncertainties.

     length=
     uncertainty=

   c. Why don't you need to determine the mass of the pendulum bob?

   d. Time a single period of the pendulum using the stopwatch.

   e. Record the measured period on paper and repeat the procedure until each group member has measured the period ten separate times.

   f. Open the Logger Pro file "Lab1.cmbl" and enter all of the results recorded by your group in the data table, under the column labeled TS. If there are three of you in the group, there should be a total of 30 entries in the column. (If the program prompts you about the sensor setup, click on "Use File As Is.")

   g. Create a histogram of the column TS. Try to make a reasonable choice for the bin width. You don't want most of the bins to be empty or have only one data point, but neither do you want the bins to be so wide that almost all of the values are in just one or two bins.

   h. From the histogram, click on the Statistics button to **calculate the statistical mean and standard deviation of your 30 data points.**

     mean =
     std deviation =

   i. Try fitting a Gaussian to the histogram by clicking the Curve Fit button. In the box for "General Equation," scroll to the bottom and select PS2 Gaussian. This function has three parameters: the amplitude A (which we don't care about), the mean M, and the standard deviation S. Before continuing, make sure that the Gaussian curve drawn by Logger Pro **reasonably** describes the data in your histogram. You are much smarter than the computer--every time you fit a curve to some data you should check that the curve looks sensible.

   What are the mean and standard deviation of the best-fit Gaussian? (Note: you might find that the program calculates a best-fit value of S which is negative. If so, ignore the minus sign; only S^2 enters into the equation for the Gaussian, anyway.)

     mean =
     std deviation =

   j. If these values are not similar to the statistical mean and standard deviation you previously calculate, or if you are having other problems getting a Gaussian fit, you may need to adjust the bin settings for your histogram.

k. Paste a screenshot of your histogram and Gaussian fit in the space below:

l. Based on these 30 data points, **what is your best estimate of the pendulum period? What is the uncertainty of this estimate?** Don't forget to specify the correct units in both of your answers.

period =
uncertainty =

m. Above you calculated the uncertainty in your period measurement via LoggerPro (forming histograms, fitting the data to a Gaussian, etc.). This process, although not very difficult, is not very intuitive. We can estimate a "worst-case" standard deviation by taking half of the *range* of our data, where the range R is defined as R = (largest data point) - (smallest data point). Similarly, we can estimate the standard error by

$$ SE \approx \frac{R}{2\sqrt{N}} $$

How does this estimated uncertainty compare to your calculated uncertainty above? Does this seem like a reasonable estimate?

For the remainder of the lab, feel free to use the estimated value for uncertainty. Having an intuitive "worst-case" value for uncertainty is better than having no value at all.

n. What are the possible sources of uncertainty in your time measurements?

Which of these contributes the <u>most</u> to the overall uncertainty in your time measurements?

o. Save your work in both NoteBook and Logger Pro before moving on to the next part.

3. Measuring one period, numerous times—Photogate

a. With the pendulum at rest, position the photogate so that the bob rests between the two arms of the photogate.

b. When you are ready to collect data, set the pendulum into motion and click the [▶] Start button to collect data.

c. Data collection will run about 1.5 minutes. Make sure you have collected data for 30 periods.

d. As before, **make a histogram and use the** [1/2] **Statistics button to determine the mean and standard deviation.**

mean=
std deviation=

Does this agree with your answer from the stopwatch portion of the experiment?

e. Paste a screenshot of your histogram and Gaussian fit in the space below:

f. What is the period and uncertainty of the pendulum as determined by the photogate?

period=
uncertainty=

Rewrite your period and uncertainty of the pendulum as determined by the stopwatch?

period=
uncertainty=

How do the values determined by the two methods compare? Why is there a difference?

g. Save your work in both NoteBook and Logger Pro before continuing.

4. Go around the room and measure the period of the other five pendulums using your stopwatch method. Be respectful of other groups still working on the first sections of the experiment. For these pendulums only measure the period one time each, rather than ten. **Based upon your answer above regarding the sources of uncertainty in your stopwatch measurements, what can we assume about the uncertainty of these time measurements? Why?**

a. Measure the mean period and length for each of the remaining five pendulums. (Numbering is arbitrary.)

                    Pendulum #2

period=   length=
uncertainty=   uncertainty=

                    Pendulum #3

period=   length=
uncertainty=   uncertainty=

                    Pendulum #4

period=   length=
uncertainty=   uncertainty=

                    Pendulum #5

period=   length=
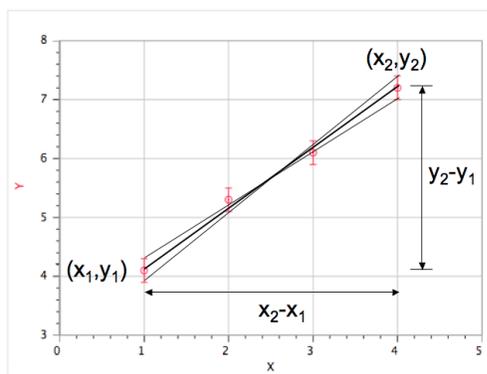uncertainty=   uncertainty=

                    Pendulum #6

period=   length=
uncertainty=   uncertainty=

b. Remember that we are looking for the standard *error* of these measurements when we talk about uncertainty.

c. Save your work in both NoteBook and Logger Pro before continuing

5. Go on to the next page entitled "Linear Regression" in the Logger Pro file. Here you will create a scatter plot with error bars of your data.

a. Enter your length and period data for the six pendulums in the correct columns of the data table. The

values for "period^2" should automatically be calculated. If they are not, ask your TF for help.

b. Enter the uncertainty for your length and period measurements in the columns marked "deltaL" and

"deltaT", respectively. Again, the "deltaT^2" column should automatically be calculated.

c. A plot of T^2 vs. L should appear in the graph to the right of the table, with error bars on all of the

points. Note that the error bar on T^2 for the pendulum you measured 30 times is smaller than the

error bars on the other points. Now click the "linear fit" button [image]. Logger Pro will calculate and
draw a line of best fit through your data points.

d. Paste a screenshot of your scatter plot and line of best fit below:

e. As with individual timing measurements, we will *approximate* the uncertainty associated with our linear
fit parameters. A simple way of doing this is to draw the steepest and shallowest lines that are
consistent with the error bars on the points. If there are vertical error bars but no horizontal error bars,
then those lines might look like this:

Using the range R as defined earlier, we can approximate the uncertainty in the slope as

$$\delta m \approx \frac{m_{max} - m_{min}}{2}$$

. You can find a similar approximation for the uncertainty in the intercept.

What are the slope and intercept of the line of best fit (with uncertainties)?

slope=
intercept=

f. As stated in the prelab assignment, pendulums can be used to calculate the value of g, the acceleration

due to gravity. **Using your results from dimensional analysis and your plot of T^2 vs. L, calculate**

a value for g with uncertainty. (The proportionality constant in the dimensional analysis is $(4\pi^2/g)$).
HINT: You will need to perform some error propagation to correctly answer this question.

g. What is the advantage of calculating g from your line of best fit over, say, calculating it from one data point?

C. Please have every group member fill out the survey at the following URL before submitting your lab report:

http://zoomerang.com/Survey/?p=WEB229M6GQ7BZE

VI. Conclusion
A. Congratulations! You've reached the end of the first lab.
B. There is a helpful sheet explaining how to submit your lab report at each station.

VII. Pre-Lab Assignment from the Problem Set
A. (These questions are here for reference.) Before doing this problem, read through the lab handout and complete the recommended Logger Pro tutorials (#1, 5, 7, 8, 9).
B. A quantity that we will encounter often in the study of motion is acceleration, which is defined as the rate of change of velocity.

1. In SI units, acceleration is measured in meters per second squared (m/s^2). What are the dimensions of acceleration?

2. In lab next week, we will study pendulums in order to calculate an important quantity called the acceleration due to gravity, or g. Suppose you measure the period T of a pendulum. The period might depend on g, as well as the mass m and length L of the pendulum. Using dimensional analysis, derive a proportionality expression for T in terms of g, m, and L.

3. Using your answer to part b), what happens to the period if you double the mass of the pendulum but keep the length constant? What if you instead double the length but keep the mass constant?