Lab 2: Conservation of Momentum

I. Before you come to lab...

A. Read through this handout in its entirety.

B. In Logger Pro, do the Tutorial named 12 Video Analysis (in the Tutorials folder under Experiments).

C. Complete the pre-lab question as part of HW 2, and turn it in with the rest of your HW by 9:30 am Tuesday.

D. Watch the following YouTube video on the Gauss gun (there's also a link from the course website):
http://www.youtube.com/watch?v=zZmCJ5eZlmo

II. Background

A. Rounding and significant figures

1. A little boy and his mother walk into the natural history museum and see a huge dinosaur skeleton in the lobby. "Wow!" says the boy. "I wonder how old those bones are?"
A nearby security guard answers, "That skeleton is 90,000,006 years old!"
"90 million *and six*? Are you sure?" asks the boy's mother.
"'Course I am," replies the security guard. "I asked the curator how old it was when I started working here, and he said 90 million years. And I've been working here for six years."

2. Okay, it's a silly joke, but it gives us pause for thought. What's wrong with the security guard's answer? The uncertainty in the age of the skeleton is on the order of millions or tens of millions of years. So it's preposterous to make any claims as to the ones digit; the six years don't matter in the least. Even if we don't necessarily quote uncertainties when tossing numbers around in everyday conversation, it's still absurd to make such a huge error in the number of significant figures.

3. Arguably, it's even more absurd to make such an error when you are, in fact, quoting the uncertainty along with the value. For example, on lab reports in the past, students have made claims such as:
"We measured a value of 1.4969461 ± 0.27777777."
If the uncertainty in the value was about 0.28, why would you claim to know the result to seven decimal places? Just because that's what your calculator said when you plugged in the numbers? Don't believe everything that your calculator tells you! Keep in mind what the numbers mean—and what they don't mean. In this case, no matter what your calculator says, you know full well that you have zero knowledge of even the hundredths digit of the answer, and only partial knowledge of the tenths digit.

4. Here's a rule of thumb you can rely on: **round the uncertainty to one significant figure**. Then round the answer to match the decimal place of the uncertainty. In our example above, we would just say that the uncertainty is 0.3, and the actual value would be rounded to the nearest tenth in accordance with the uncertainty. So the final result would be:
"We measured a value of 1.5 ± 0.3."
If the uncertainty were only 0.0027777, we'd call it 0.003 and the result would be 1.497 ± 0.003. The small uncertainty gives us confidence that the thousandths digit is meaningful.

5. One exception to the rule of thumb: If rounding the uncertainty to one significant figure would cause that figure to be a 1, then you keep the next digit as well. So for instance, if you have 5.83333333 ± 0.14285714, then you would round the uncertainty to 0.14 (instead of just 0.1) and report 5.83 ± 0.14 (instead of 5.8 ± 0.1).

B. Momentum

You've already learned about momentum in the first few weeks of lecture, section, and homeworks. But here is a summary of the big ideas:

1. The total momentum of an isolated system is conserved.

- **a.** The **momentum of a single object** is its mass times its velocity. Momentum is a vector.
- **b.** The **total momentum** of a system is the vector sum of the momenta of each object in the system.
- **c.** An **isolated system** is one that has no interactions with anything outside of the system.
- **d.** A system can be considered to be **functionally isolated** if there is no net external force on the system.

2. The **impulse** on a system is defined as the *change* in the system's total momentum between some initial state and some final state (final momentum minus initial momentum).

- **a.** For an isolated system, therefore, the impulse is zero no matter what the initial and final states are chosen to be.
- **b.** For a non-isolated system, the impulse on the system during some interaction is given by the average external force on the system multiplied by the time duration of the interaction.

III. Introduction

A. Scientists are rarely given explicit procedures when they walk into their labs every morning. They have a problem they want to solve and posses a figurative toolbox of concepts, equations, and techniques, in addition to an *actual* toolbox of available equipment, to help them do so. They will determine what phenomena to investigate, what values to measure, and design an appropriate procedure—an experiment—to follow in the lab.

B. With this in mind, you will need to design a procedure for this lab. You will explore the conservation of momentum in an interesting physical system: the Gauss gun. As you saw in lecture and in the online videos, the Gauss gun can be used to shoot steel ball bearings at high speeds. The details are complicated because they involve magnetic forces, which we won't cover in this course. However, the beauty of momentum conservation is that we don't have to know or care about the details of the interaction: we just look at the initial momentum and the final momentum and compare them. You will design an experiment to answer the question: **"Is momentum conserved during the firing of the Gauss gun?"**

C. Learning objectives for this lab:

- **1.** Investigate momentum conservation
- **2.** Learn how to determine the combined effects of many different sources of uncertainty in a single calculation
- **3.** Learn how to report correctly rounded answers

IV. Materials

A. Magnet, 3 steel ball bearings, and some track

- **1.** What they are: a magnet, 3 steel ball bearings, and some pieces of track.
- **2.** What you can do with them: make a Gauss gun! Check it out! http://youtu.be/ZR_ctC287Gk
- **3.** A Gauss gun is an unusual physical system that we can't yet fully describe, because don't yet have a quantitative model for magnetic forces. However, if we take the physical system to be the entire Gauss gun (magnet plus all 3 balls), then we can still describe the *external interactions* of the system.

B. Carbon paper and typing paper

- **1.** What they are: sandwiched sheets of carbon paper and regular white typing paper, taped to the floor.
- **2.** What you can do with them: If you drop a ball onto the floor where they're taped, it will leave a mark on the white paper exactly where the ball hit.

C. 2-meter stick

- **1.** What it is: like a meterstick, except twice as long.
- **2.** What you can do with it: measure lengths up to 2 m.

D. Digital video camera on tripod

- **1.** What it is: pretty sweet.

- **2.** What you can do with it: record video of something at 30 frames per second. Then, you can measure the motion of objects in the video using Logger Pro's video analysis capabilities.

E. Lubricant

- **1.** What it is: slippery.
- **2.** What you can do with it: reduce sliding friction by applying a small amount of it to the track.

F. Level

- **1.** What it is: a carpenter's level.
- **2.** What you can do with it: make sure that a surface is really horizontal. If it's not, you can shim one side or the other with pieces of paper.

G. Balance

- **1.** What it is: a digital balance.
- **2.** What you can do with it: measure the mass of small objects.
  - **a.** The balances read to the nearest hundredth of a gram. That means that when you measure a mass using the balance, the reading error on the mass is 0.01 g.
  - **b.** If you use the balance to measure the mass of a magnet, you might get an inaccurate reading because of an attractive force between the magnet and the metal inside the balance. To get an accurate measurement, put an inverted plastic cup on top of the balance and re-zero it before putting the magnet on top. That way the magnet will be several inches away from anything metal while it is being weighed.

H. Computer with Logger Pro installed on it

- **1.** What it is: what it sounds like.

2. What can you do with it: data collection and analysis.

V. Procedure

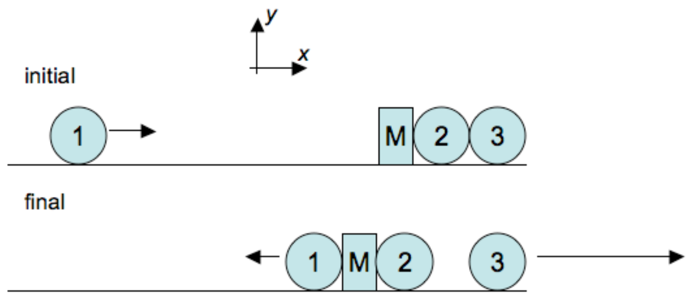
A. Before you begin:

1. Take a picture of yourselves using Photo Booth and drag it into the space below:

2. Tell us your names (from left to right according to the above photo):

B. Measurements

1. The goal of today's lab is to observe the Gauss gun system and see whether its momentum is conserved during the brief but violent magnetic interaction. Let's start by defining the system:



a. There are three balls (1, 2, and 3) and a magnet (M). Initially, ball 1 rolls to the right (in the +*x* direction), while the M-2-3 trio are stationary.

b. Then the big magnetic interaction occurs. However, if the system is defined as 1+2+3+M, then that big magnetic interaction is an *internal interaction*, so it does not change the total momentum of the system.

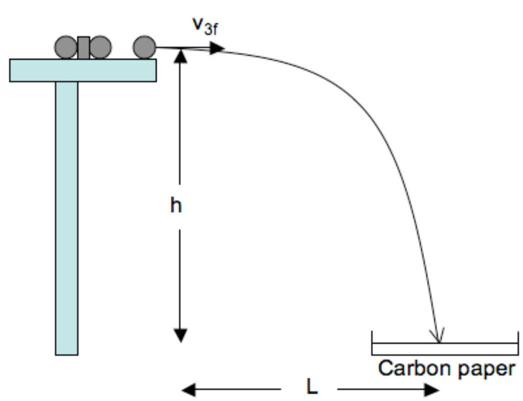
c. In the final state, ball 3 shoots off to the right very fast, while the 1-M-2 trio recoil to the left somewhat slowly.

d. We'll be analyzing the interaction using the video camera, which is limited to 30 frames per second. It is likely that we will not be able to capture the exact instant of the collision in the video, because it will occur between frames. So let's make a specific definition: for the purposes of this

(1) "initial" refers to the *last video frame before the magnetic interaction between ball 1 and the magnet*

(2) "final" refers to the *first video frame after the collision*

e. In order to talk about momentum, we'll have to measure some masses (easy), and three different velocities (harder): the initial velocity of ball 1, the final recoil velocity of 1-M-2, and the final velocity of ball 3. The first two will be calculated from the video. However, ball 3 shoots out too fast to capture in the video. Instead, we'll figure out how fast it is launched using the following setup:



The track is positioned so that it ends right at the edge of the lab bench. Ball 3 will be launched off the end of the track, and it will strike a piece of carbon paper on the floor, leaving a dark mark. If we measure the height of the bench h and the horizontal distance L from the end of the track to the mark on the paper, we can calculate the velocity with which the ball was launched (assuming it is initially moving in a purely horizontal direction).

2. Take a few minutes to play around with the setup and plan how you will arrange the various pieces (video camera, balls, magnet, track, carbon paper, box to catch ball 3 after it bounces off the floor). Make sure the entire track is visible in the video camera display. Do a few test runs before you start taking any data. You'll only have time to analyze a single run, so you'll want to make sure everything is set up correctly. A very small amount of lubricant (just a drop or two) on the track will help reduce friction.

3. The last section of the Lab Handbook has a very useful set of "Experimental Hints" on how to use the video camera to insert a video capture into a Logger Pro file. When you are ready to record, make sure that everything is set (fresh sheet of white paper under the carbon paper, box in place, etc.) before you click Start Capture. Then roll ball 1 slowly towards the magnet, and watch the collision closely.

4. When the video capture finishes, make sure all of the following things are true:

- **a.** Ball 1 approaches the magnet smoothly (no bouncing or wobbling as it rolls)
- **b.** Collision does not impart any significant y-velocity (vertical motion) to any part of the system (no bouncing)
- **c.** Recoiling 1-M-2 system comes to rest *on the track*, and the *entire recoil* (until everything comes to rest) is recorded in the video capture
- **d.** You can clearly identify which carbon paper mark was the one left by the ball during your actual run

5. If any of the above are not true, delete the video capture and repeat the experiment. It doesn't take long to do, but it does take a lot of analysis, and you don't want to waste your time analyzing data from an incomplete run.

6. Remember, you *don't* need to be able to see ball 3 shoot off in the video capture. We expect that you won't, actually, because it goes too fast. That's why we have the carbon paper setup.

C. Analyzing the data

1. At some point either before or after your video capture, measure the masses of the ball and the magnet. It's reasonable to assume that all three balls are identical. Record your values, with uncertainties, here:

mass of one ball:
mass of magnet+2 balls:

2. Initial state

a. The only object with momentum in the initial state is ball 1. Use the video analysis tools to measure **v1ix**, the *x*-velocity of ball 1 in the initial state (*before* it speeds up suddenly due to its attraction to M).

(1) Remember, you'll have to set the scale for the video. Use the length of the track itself to set the scale.
How long is the track? (Measure it with a ruler.)

(2) You might need to tilt your axes to make sure all the motion is in the x-direction. This can happen if your video camera is not aligned perfectly, so that the track is exactly horizontal. Once you have done this, we are no longer interested in the y-motion (there shouldn't even *be* any y-motion), so change the settings on the graph so that it *only shows you x vs t*. (We also don't want it to show vx or vy. You'll make your own determination of velocity.)

(3) Use a linear fit to an appropriate section of the x-vs-t graph to determine the velocity. **Paste your graph and record your value here**:

v1ix =

b. To get the uncertainty of this value, we need to do two things:

(1) First, estimate the uncertainty of the x-values obtained from the video analysis. You can use the same technique you used in the prelab question. **Record your measurements and estimate the uncertainty in x here**:

(2) Second, assume that the error bars on your x-vs-t graph have the size you calculated in part (1). Using the same method you used in Lab 1, calculate the uncertainty in the slope that results from error bars of this size. **Record your answer here**:
uncertainty of v1ix =

c. Save your work here and in Logger Pro.

3. Final state

a. Recoil of 1-M-2

In the same video capture, you can track the motion of the recoiling 1-M-2 trio after the collision. However, unlike the incoming ball 1, we do not expect that 1-M-2 moves at constant velocity after the collision. It has some velocity right after the collision, but later it is observed to come to rest. So there must be a net force acting on it.

(1) Using the video analysis tools, plot the position of the 1-M-2 trio from the first frame after the collision (which we'll call tc) until it comes to rest (at time tr).

(2) The best model of the forces acting on the recoiling trio predicts that it will undergo *constant acceleration* from time tc to tr. (We'll see why this is the case next week.) **If the motion really is constant acceleration, what kind of curve will describe the x-vs-t graph?**

(3) Try fitting such a curve to the relevant time interval in your video analysis data set. Write the equation so that one of the fit parameters is tr. This is better than guessing a value of tr from the video, since tr will likely occur between successive frames. (tc, of course, falls exactly on a video frame, by definition.) If you're not sure how to do this, ask your TF. **Does the curve look like a good fit for your data?**

Paste the graph, with fit, here:

(4) Assuming the curve looks like a good fit, we won't even use the values of the parameters it gives us, except for tr. Instead, we can get the quantity we are interested in (the instantaneous x-velocity at time tc) from the equations for motion with constant acceleration. The two quantities you can get directly from the video analysis are Δx, the change in x-position, and Δt, which is just tr minus tc. **What are the measured values of these quantities?** (NB! Δx should be *negative*, because 1-M-2 moves to the *left* during the recoil.)

Δx =
Δt =

(5) What is the uncertainty of Δx?
uncertainty of Δx =

Where did this number come from?

(6) What is the uncertainty of Δt?
uncertainty of Δt =

Where did this number come from?

(7) Now let's solve a physics problem. You'll probably need pencil and paper for this. **Assuming motion with constant acceleration, what was the x-velocity of 1-M-2 at time tc, in terms of the two quantities Δx and Δt?** You don't have to show your work here, but once you have an answer, check with a TF to make sure it is right before proceeding.

(8) Plug in your numbers from part (4) to calculate a numerical value for the x-velocity of 1-M-2 just after collision, which we'll call v1fx:

v1fx =

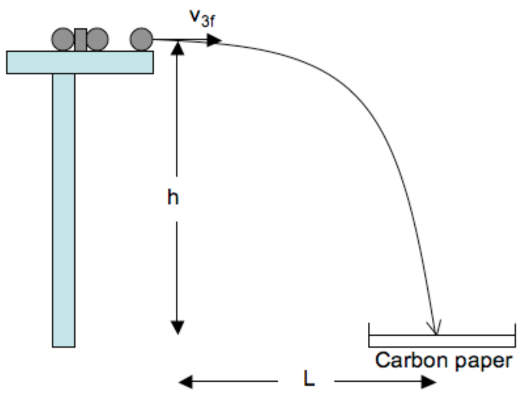
(9) Use the "worst-case scenario" error propagation method to calculate the uncertainty in v1fx from the uncertainties in Δx and Δt:

uncertainty of v1fx =

(10) Save your work here and in Logger Pro.

b. Ball 3 ("bullet") launched off the table

(1) Using the carbon mark left by ball 3 when it hit the floor, measure the quantities h and L from the figure (repeated below for convenience). Include an estimate of your reading errors.



h =
uncertainty of h =

L =
uncertainty of L =

(2) Time to solve another constant-acceleration problem: **In terms of h, L, and g, how fast was ball 3 going when it left the table?** Again, you don't need to show your work, but check your answer with a TF before proceeding.

(3) Plug in your numbers from part (1), and the accepted value of g, to get the x-velocity of ball 3 just after the collision:

v3fx =

(4) Again, use error propagation to calculate the uncertainty in v3fx based on your estimated uncertainties in h and L. You may neglect any uncertainty in g.

uncertainty of v3fx =

(5) Save your work here before moving on.

4. Putting it all together

a. Calculate the x-component of the total initial momentum, with uncertainty:

p_ix =
uncertainty =

b. Calculate the x-component of the total final momentum, with uncertainty:

p_fx =
uncertainty =

c. What can you conclude from your data about momentum conservation in the Gauss gun system?

VI. Conclusion

A. Congratulations! You've reached the end of the lab.
B. Submit your lab report online.
C. Before you leave the lab, ask your TF to make sure that your lab writeup has been properly uploaded on Sapling.

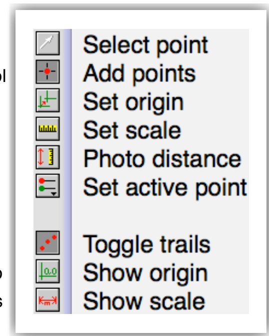
VII. Pre-lab assignment from the Problem Set

(These questions are here for reference.) *Before doing this problem*, read through the lab handout for Lab 2 (from the course website), and work through the Logger Pro tutorial named 12 Video Analysis (in the Tutorials folder under Experiments). Then answer the following questions about the tutorial.

A. Let's call t_i the time that Dave releases the basketball, and t_f the time when it reaches the peak of its trajectory. Using your analysis of the ball's position, measure both the time interval $\Delta t = t_f - t_i$ and the height change $\Delta y = y_f - y_i$ that the ball underwent during that time. Hint: if you find it difficult to estimate values off the graph, you can insert a data table with the raw numbers by going to Insert → Table. Alternatively, use the Examine feature from the toolbar, which will show you the numerical coordinates of the nearest data point as you mouse around the graph.

When you perform a measurement using video analysis, it's just like any other measurement: you need to know the uncertainty as well as the value. The uncertainty in the value of Δt is pretty easy to estimate as a reading error: the timing device is just the internal clock of the video. Assuming it's calibrated correctly, the uncertainty in any time measurement made using the video is just the interval between frames. This video was shot at 30 frames per second, so the uncertainty in Δt is about 1/30 of a second, or 33 ms. (It would also not be unreasonable to claim that the reading error was half this much, if you think you can confidently make a statement like, "the ball reached its peak closer to this frame than the next frame.")
However, the uncertainty in Δy is subtler. There is obviously reading error in using the video analysis procedure to measure the position of the ball. Since the measurement is made by using your mouse to click on the ball's position, one reasonable estimate is that the reading error is on the order of one pixel. However, there is also error introduced in the calibration step: we are approximating a length that we call 2 meters, and use that length to calibrate all other distance measurements. We can estimate this calibration error the old-fashioned way: by repeating the "measurement." You can't set the scale for the video multiple times, but you can use the "Photo Distance" tool to simulate repeating the measurement.

B. So let's do it. Here's the procedure: first, uncheck the box for "Show scale" using the bottom button in the video analysis toolbar. (A screenshot of the toolbar buttons and what they do is included at right.) The green line showing your 2-meter scale will disappear. Next, use the "Photo distance" tool (ruler with a double-headed red arrow). When you click the button, you can then click and drag between two points in the video and Logger Pro will calculate the distance for you, using the scale you've already set. This process is exactly like setting the scale, so we'll use it to "repeat" the measurement. Use the photo distance tool to "measure" the length of the two-meter stick several times, and report your values. Of course, they will all be approximately 2 meters, but what we are interested in is the *range* of values for the length of the stick. Are they between 1.95 and 2.05 m? 1.99 and 2.01 m? 1.999 and 2.001 m? The spread tells you the reading error when using the video analysis to measure a distance of 2 m. And in fact, since we assume that the reading error in pixels is always about 1 pixel, the value can be used as the absolute reading error in any measurement made from the same video, including Δy. What is this value?



C. According to constant-acceleration kinematics (more on this in the next homework), Δt and Δy should be related by the equation

$$\Delta y = \frac{1}{2} g (\Delta t)^2$$

where g is the acceleration due to gravity. Using your measured values of Δt and Δy, as well as the uncertainties you estimated in part b), calculate a value of g as determined by the basketball measurement, *with uncertainty*. (You will have to do some error propagation—recall the "worst-case" method from Lab 1.) How does it compare to the accepted value of 9.81 m/s^2?