

Lab 4: Work and Energy

I. Before you come to lab

- A. Read through this handout in its entirety.
- B. Complete the pre-lab question as part of HW5, and turn it in with the rest of your homework by 9:30 am Tuesday.

II. Introduction

- A. Objective for this lab:
 - 1. Learn how to relate the new concepts of work and energy to concepts you are already familiar with (force and distance)
 - 2. Learn how to formulate an explanation for a phenomenon - how the Gauss Gun fires - and quantitatively test that explanation through experiment and data analysis
- B. This lab is about work, and how work connects force and energy. As a playground for these ideas, we will continue to explore the Gauss Gun system we saw a few weeks ago. The interaction of the magnet and balls is in some ways a "mystery" interaction because we don't know what the force law is or what the potential energy is. However, we can tell that there is energy associated with that interaction, since it appears that the magnetic interaction can be a source of kinetic energy. Even though we don't have a way of measuring energy directly, we can use our understanding of work to get at the potential energy of the magnetic interaction by measuring the magnetic *force* at different distances. Once we know the energies, we can explain why the kinetic energy changes so much in the operation of the Gauss gun.

III. Background

- A. Error propagation: Advanced techniques

The extra handout on error propagation introduces new techniques and equations for doing error propagation problems. By understanding and using these techniques, you can save yourself quite a bit of calculation. The uncertainties calculated using these techniques are also generally a bit more realistic and a bit smaller than those calculated using our old "worst-case" method.

 - 1. Recall that our existing method is to "crunch three times": once to get the mean (middle) value of the final answer, once to get the largest possible value, and once to get the smallest possible value. Then the mean gives the value, and $(\text{largest} - \text{smallest})/2$ is the uncertainty.
 - 2. By using the new techniques instead, you still have to crunch once to get the value (mean), but now you only have to do one more calculation to get the uncertainty. Sometimes the expression for the uncertainty can look complicated or daunting, but there are useful approximations that we can use, relying on the fact that we can only really need to know the uncertainty to about one sig fig.
 - 3. A word of warning: many of the new error propagation formulas are most easily used and remembered if they are stated in terms of *fractional uncertainty* (or percent uncertainty), instead of absolute uncertainty. Make sure you are comfortable with both and can recognize the difference. As an example, if I measure a length to be 2.00 ± 0.12 m, the uncertainty in the length is 0.12 m (or 12 cm), but I could also say that the length is 6% uncertain (with no units), because 0.12 m divided by the value of 2.00 m is 0.06.
 - 4. Most importantly: Starting with this lab, **all error propagation is to be done using the new techniques if at all possible**. There are three main ways in which the new techniques might be inapplicable:
 - a. when your calculation involved *correlated errors*. It is often, but not always, possible to re-write your equation so that the errors become uncorrelated. If you can't, you'll have to fall back on the worst-case method
 - b. when your calculation involves some mathematical function other than add/subtract/multiply/divide/raise to a power, e.g. sin or exp or log. For instance, if you measure an angle to be $30^\circ \pm 2^\circ$, what is the uncertainty in the sine? Again, you can fall back on the worst-case method: $(\sin 32^\circ - \sin 28^\circ)/2$.
 - c. when your calculation involves a numerical integral or derivative of some data points. We'll see this in action during today's lab.
 - 5. If you are curious about the derivations of these equations from Gaussian statistics, you can look them up in chapter 5 of Taylor.
- B. Potential energy

1. Potential energy is the stored energy of an interaction. The key identifying characteristic of potential energy is that it can be converted into kinetic energy by the force associated with the interaction.
- The most familiar example of potential energy is gravitational potential energy. An object of mass m in the gravitational field of the earth has gravitational potential energy equal to $U_{\text{grav}} = mgh$, where h is its height. This means that in order to raise an object by a height h , you need to supply it with an amount of energy equal to mgh ; conversely, letting it fall a distance h releases mgh of potential energy. (In free-fall, this would all be converted into kinetic energy of the object.)
 - Another example is elastic potential energy, the potential energy stored in a spring or other elastic material. If a spring has spring constant k , then the potential energy stored in the spring is equal to $U_{\text{elastic}} = 1/2 k x^2$, where x is the distance by which the spring is stretched from its relaxed (neutral) length. (x is negative if the spring is compressed.) If you stretch a spring, the spring has elastic energy, which can be converted into some other form of energy (e.g. kinetic) through the action of the spring force: the spring will try to return to its relaxed length and will exert a pulling force on whatever is attached to it in order to do so.
- c. One important fact about potential energy is that **there is no absolute measurement of potential energy**. Instead, only *differences* of potential energy between two configurations can be measured. This means that any time you express potential energy as a function of position (for example, $U_{\text{grav}}(h) = mgh$ or $U_{\text{elastic}}(x) = 1/2 kx^2$), you could add an arbitrary constant to the potential energy without affecting the physical meaning of the equation.
- (1) For example, if you are dealing with the gravitational potential energy of a 1-kg mass, you could say that $U=0$ when the mass is on the ground. If you then lift the mass onto a table 1 m high its new gravitational potential energy is $m=1$ kg, times $g=10$ m/s², times $h=1$ m, which gives $U=10$ joules. However, you could also say that $U= -10$ J when the mass is on the ground, and 0 J when it is on the table. (This amounts to defining your $h=0$ to be at the height of the table.)

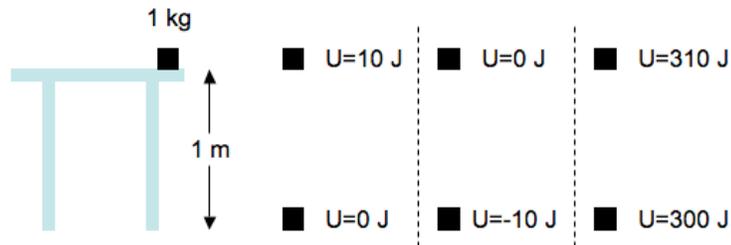


Figure: Three equivalent ways of defining gravitational potential energy

You could even say that $U=300$ J at ground level and $U=310$ J at table level. It doesn't matter. The only relevant physical quantity is the difference in potential energy between the floor and the table.

- (2) Only the *difference* between potential energies has any physical meaning:
 $U(B) - U(A) =$ amount of energy required to move the object from A to B. If this quantity is positive, it means that the potential energy is higher at B, and therefore you need to add energy to the system in order to go from A to B. (As we will see shortly, the way you would accomplish this is by doing *work* on the system.) If it is negative, it means that you would get extra energy by letting the object go from A to B.

C. Work and force

- A force that acts on a moving object does work on that object. In general, work is defined as the dot product of the (vector) force and (vector) displacement of the object. But if the force and motion are confined to *one dimension*, we can use a simpler expression:

The work done by a force F is equal to the (scalar) force F multiplied by the (scalar) displacement dx .

If the force and displacement are in the same direction, the work done by F is positive; if they are in opposite directions, the work done by F is negative.

- If the force is not constant during the object's motion, we can use calculus to get the work. As usual, we divide up the motion into very short segments and approximate the force as being constant over each segment. Then we just add up all the little bits of work, $F dx$ for every little dx . This is represented as an integral: the work done by F as an object moves from $x=a$ to $x=b$ is equal to

$$W = \int_a^b F(x) dx$$

If there is more than one force acting on the object, then the net work is equal to the algebraic sum of the work done by each force.

▼ D. Work and energy: 3 important results

- 1. In lecture, we saw the *work-kinetic energy theorem*, which states that the net work done on an object (that is, the sum of the work done by every force acting on it) is equal to the change in that object's kinetic energy. We can also consider the net work as the sum of the work due to conservative forces and the work due to non-conservative forces:

$$W_{net} = W_{Conservative} + W_{Non-conservative}$$

- 2. If a force F is *conservative*, then there is a potential energy function $U(x)$ associated with it, such that

$$F = -\frac{dU}{dx}$$

Plugging this into the equation for the work done by a force, we get

$$W_{conservative} = \int_a^b \left(-\frac{dU}{dx} \right) dx = -\Delta U$$

In words, **the work done by a conservative force in moving from one point to another is equal to minus the change in potential energy.**

- 3. If there are both conservative and non-conservative forces, we can combine the two results above. The total work is equal to the work done by conservative forces, plus the work done by other forces. By the work-KE theorem, this should be equal to the change in the object's kinetic energy; but using the equation for work done by a conservative force, we can get the *work-energy theorem*:

$$W_{non-conservative} = \Delta K + \Delta U$$

The change in an object's total mechanical energy is equal to the work done on it by non-conservative forces.

Be sure you understand the difference between the work-KE theorem and the similarly-named work-energy theorem. The work-KE theorem equates the change in an object's *kinetic energy* to the total work done on it by *all forces*; the more general work-energy theorem equates the change in an object's *total mechanical energy* to the work done on it by *non-conservative forces*.

▼ IV. Materials

▼ A. Force sensor

- 1. The force sensor is a small box with a hook that can measure the forces applied to the hook. It is mounted on a horizontal bar.



- 2. The force sensor connects to the computer via the LabPro interface and the data can be read into a computer using the Logger Pro software. The sign of the force reading is considered to be positive if you pull on the hook and negative if you push on it.
- 3. There is a switch on the force sensor which toggles between force ranges of ± 10 N and ± 50 N. The ± 10 N setting has higher sensitivity and is more desirable to use most of the time, but if you are measuring forces larger than 10 N, obviously you will have to use the ± 50 N setting. When you flip the switch, Logger Pro will ask you to either change it back or use the sensor setting; click the option to use the sensor setting.

B. Lab jack

- 1. This is a simple piece of equipment that enables you to adjust the height of a small platform.



- 2. The height of the jack can be controlled by turning the knob on the side.

C. Spring and mass stand with hook

- 1. The spring is just a coiled spring. It can be stretched, but not compressed.



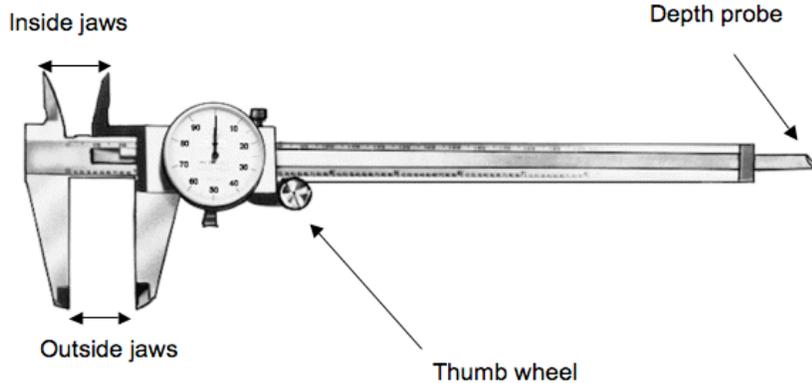
- 2. The mass stand and masses can be used to stretch the spring.

D. Magnet and steel ball bearings

- 1. These are the same magnets and ball bearings that you used to make the Gauss gun.

E. Dial caliper

- 1. A caliper is a device for measuring small distances very precisely:



- 2. The caliper has three measuring modes: outside jaws that you can use to measure the thickness of something; inside jaws that you can use to measure the size of an opening (the inner diameter of a pipe, for instance); and a depth probe that you can use to measure the depth of a hole. The jaws can be moved with the thumb wheel until they fit the distance being measured. The linear scale along the caliper indicates the length roughly, and the dial indicates the length much more precisely.
- 3. For the calipers we will be using in the lab the dial spins once every 2 mm, and the smallest tick mark on the dial corresponds to a change of 0.02 mm.

F. Shim stock

- 1. These are simply pieces of aluminum or plastic cut to a certain thickness. (Unlike steel, aluminum does not affect the magnetic interaction.)

V. Procedure

A. Before you begin:

- 1. Take a picture of yourselves using Photo Booth and drag it into the space below:

- 2. Tell us your names:

B. Playing around with the force sensor

- 1. Open the file Lab4.cmb1 in Logger Pro.
- 2. Set the force sensor to the ± 50 N range.
- 3. You should see a digital meter in the lower left which is the reading on the force sensor. Watch what happens when you push or pull on the hook of the sensor. What if you hang something from the hook? Which direction gives positive force readings, pushing or pulling?



- 4. Try clicking the Zero button. What happens? Pull on the hook and click it again. What now? (You don't have to write answers to these questions; they're just things for you to notice and learn.)



- 5. Click the Collect button and watch what happens to the Force vs Time graph on the right. Push or pull on the hook while it is collecting. The collection runs automatically for 10 seconds. You can, of course, just click the button again to collect more data.

- 6. When you are satisfied with your understanding of the force sensor, you can move on.

C. Work done by a spring

- 1. Set up the force sensor by flipping the range switch to the ± 10 N setting. If Logger Pro complains about this change, click "Use the sensor setting" instead of "OK."

- 2. We would like to use the force sensor and other equipment (heavy mass, lab jack, and meter stick) to calculate the

work done by the spring as it stretches from $x = 0$ to 4 cm. You'll need to think carefully about how to do this.

- a. The springs we are using can only stretch, not compress, because the "equilibrium" state is fully compressed. To avoid having to deal with the equilibrium state, choose a position that involves the spring being very slightly stretched, and call that $x = 0$.
 - b. **Given the materials listed above, come up with a procedure to measure the work done by the spring on the mass over a range of 4 cm, and explain it here:**
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- 3. **Make a graph of the force exerted by the spring versus distance and paste it below. (Scale the graph so that the relevant part of it is showing.)**
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- ▽ 4. Graphically calculate the following quantities. (Don't worry about uncertainties.)
 - a. **The work required to stretch the spring from 0 to 2 cm**
 - b. **The work required to stretch the spring from 2 cm to 4 cm**
 - c. **Are these equal to each other? Why or why not?**
 - 5. **Calculate the work done *by the spring on the mass* in stretching from 0 to 4 cm.**
 - 6. Consider the potential energy U_{spring} . **What is the difference in U_{spring} between the initial (less stretched) position of the spring and its final (more stretched) position?**

In which position is the potential energy greater?

- 7. Save your work in both Logger Pro and NoteBook before moving on to the next part.

▽ D. Force exerted by a magnet

We can use the force sensor, with a clever setup, to characterize the magnetic force between the magnet and the ball. The one thing that we need to assume is that the magnetic force depends on only two things: the separation distance, and the ordering of the objects (ball-magnet-ball or magnet-ball-ball).

- 1. Turn to page 2 of the Logger Pro file.
- 2. Remove the spring from the force sensor and put the mass stand and lab jack aside. With the force sensor on the ± 10 N setting, hang the yellow string with the ball bearing from the hook of the force sensor and zero it.
 - a. **Draw a FBD for the hanging ball. Take a picture of it and place it here:**
 - b. **How does zeroing the force sensor affect your FBD?**
- 3. Take another ball of the same size and attach it to the magnet's upper surface. Let's call this ball 1.
- 4. Touch the top of ball 1 to the bottom of the hanging ball (ball2), click the  Collect button, and then gently pull the magnet block straight down. **Paste a copy of the Force vs. Time graph here and note your**

observations of the plot:

- - a. Draw a FBD of the hanging ball *during* the pull. (*Hint: ball 1 exerts both a magnetic force and a normal force on ball 2.*) Take a picture of your FBD and place it here:

 - b. From this new FBD, write an expression for the number read by the force sensor. Remember that you began by zeroing the sensor.

 - c. How can you use the plot to determine the magnetic force on ball 2?

- - 5. Click  Collect and repeat your measurement at least three or four times during the 10 seconds, and compare the values of the peak force each time. If the uncertainty exceeds a few tenths of a newton, there may be systematic error. Repeat the measurements.
 - 6. Record the values into the data table "Stats" at the bottom of the page. It will automatically calculate the mean value of the values that you enter. Record this number in the "Force" column in the table named "Magnet Ball Ball" on page 2 of the Logger Pro file, with a corresponding distance of 0 (since the two balls are touching in this configuration).
 - 7. There is also a column called "SE" that will calculate the uncertainty of your force measurement. Enter this number into the "Magnet Ball Ball" table into the column "ΔF". Logger Pro should automatically fill the F_{\max} and F_{\min} columns.

- - 8. To determine the magnetic force when the balls are separated, put the thinnest shim between the two balls so that they are separated by the thickness of the shim. Repeat the measurement as above. Measure both the force, using the method in steps 5 and 6, and the separation distance (thickness of the shim) and record the data in the next row of the "Magnet Ball Ball" data table.
 - 9. Continue repeating the measurements with consecutively thicker shims until the force finally drops to nearly zero. ***Be careful***—if you are using a rigid shim, as opposed to a floppy piece of cellophane, make sure you are not using the shim to push upwards on the hanging ball during the measurement. The shim should never be out of contact with ball 1; otherwise the thickness of the shim will not be equal to the separation distance during your force measurement. Record all of your measurements in the same data table. You should end up with about 5-8 data points total.
 - 10. Now flip the magnet upside down, so that ball 1 is below the magnet. This way, you can measure the magnetic force in the ball-magnet-ball configuration.

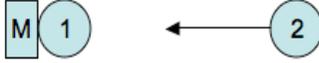
- - 11. Repeat the measurement process (steps 5-8) for this configuration, except this time you'll populate the lower data table, called "Ball Magnet Ball." *NB:* you will need to set the force sensor to the ± 50 N setting. After you flip that switch, Logger Pro will complain again (tell it "Use sensor setting"), and you'll need to zero the force sensor again with the ball hanging freely.
 - 12. You don't need to take quite as many data points at very short separation distances. Again, 5-8 data points is sufficient, so feel free not to use every single shim.
 - 13. Save your work before moving on to the next part.

- - ▼ E. Work done by a magnet
 - 1. Now turn to page 3 of the Logger Pro file.
 - ▼ 2. You'll see both of your data tables that you populated with your measurements in the previous part. **Create graphs of magnetic force vs separation distance for both configurations, magnet-ball-ball and ball-magnet-ball. Paste those graphs here:**
 - a. Magnet-ball-ball

b. Ball-magnet-ball

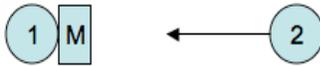
3. Using your graphs, calculate the following quantities:

- a. the **work done by the magnetic force on ball 2** as the separation decreases from very far away to zero, like so:



$W_{\text{mag} \rightarrow 2} =$

- b. the **work done by the magnetic force on ball 2** as the separation decreases from very far away to zero, like so:



$W_{\text{mag} \rightarrow 2} =$

4. Consider the magnetic potential energy U_{mag} , which depends on the separation distance and the ordering of the objects. Suppose we define U_{mag} to be equal to zero when the magnet+ball 1 are together, very far away from ball 2.

- a. To calculate the uncertainty in the work, you'll have to fall back on the old (worst-case scenario) method of error propagation. Luckily, Logger Pro has been keeping track of F_{max} and F_{min} for you.
 b. What is U_{mag} with uncertainty for this configuration? (Be careful about signs!)



$U_{\text{mag}}(\text{magnet ball ball}) =$

- c. What is U_{mag} with uncertainty for this configuration?



$U_{\text{mag}}(\text{ball magnet ball}) =$

F. Putting it together

1. Now calculate the **difference** in potential energies between the two configurations. Don't forget uncertainties!

$\Delta U_{\text{mag}} = U_{\text{mag}}(\text{magnet ball ball}) - U_{\text{mag}}(\text{ball magnet ball}) =$

Is this quantity positive or negative? Is that what you expected?

2. Can you explain, using concepts of energy, what happens during the operation of the Gauss gun and why the second ball comes out so fast?

3. Which of the following findings would surprise you when comparing the magnitudes of

$\Delta K = (K_{\text{aftercollision}} - K_{\text{beforecollision}})$ and ΔU_{mag} ? Why?

- a. $|\Delta K| < |\Delta U_{\text{mag}}|$
 b. $|\Delta K| = |\Delta U_{\text{mag}}|$
 c. $|\Delta K| > |\Delta U_{\text{mag}}|$

4. This is the end of the lab itself, but if you have time left, you may want to start not he post lab question (from HW6), which combines the results of Labs 2 and 4.

▼ **VI. Conclusion**

- A. Submit your lab report online. Don't forget to save your work as a pdf file!

○ ▼ **VII. Pre-Lab Assignment from the Problem Set**

These questions are here for reference.

- A. Look up your lab report from Lab #2 . What were your recorded masses and velocities, with uncertainties? Based on these, calculate the total kinetic energy of the system in both the "initial" (just before collision) and "final" (first video frame after collision) states.
- B. Use error propagation to determine the uncertainties in the two energies you calculated in part a). What can you say about whether kinetic energy was conserved during the interaction?

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