Lab 5: Harmonic Oscillations and Damping

▽**I. Introduction**
  ▽**A.** Objectives for this lab:
    ◯**1.** Learn how to quantitatively model a real harmonic oscillator
    ◯**2.** Learn how damping affects simple harmonic motion
  ◯**B.** In this lab, you'll explore the oscillations of a mass-spring system, with and without damping. You'll see how changing various parameters like the spring constant, the mass, or the amplitude affects the oscillation of the system. You'll also see what the effects of damping are and explore the three regimes of oscillatory systems—underdamped, critically damped, and overdamped.
  ◯**C.** Read section 12-7 in Kesten and Tauck on The Damped Oscillator.
▽**II. Background**
  ◯**A.** Harmonic motion
    Most of what you need to know about harmonic motion has been covered in the lectures and in Kesten and Tauck Chapter 12, so we won't repeat it in depth here. The basic idea is that simple harmonic motion follows an equation for sinusoidal oscillations:

$$x_{undamped} = A\cos(\omega t + \phi)$$

    For a mass-spring system, the angular frequency, ω, is given by

$$\omega = \sqrt{\frac{k}{m}}$$

    where m is the mass and k is the spring constant. Note that ω does not depend on the amplitude of the harmonic motion.
  ▽**B.** Damping
    ◯**1.** The situation changes when we add *damping*. Damping is the presence of a drag force or friction force which is non-conservative; it gradually removes mechanical energy from the system by doing negative work. As a result, the sinusoidal oscillation does not go on forever. Mathematically, the presence of the damping term in the differential equation for x(t) changes the form of the solution so that it is no longer a simple sine wave.
    ▽**2.** Kesten and Tauck Section 12-7 describes the effects of damping. Essentially, the system will be in one of three regimes, depending on the amount of damping:
      ▽**a.** Small damping causes only a slight change in the behavior of the system: the oscillation frequency decreases somewhat, and the amplitude gradually decays away over time according to an exponential function. This is called an *underdamped* system. The solution to the equation of underdamped systems is:

$$x_{underdamped} = Ae^{-\lambda t}\cos(\omega' t + \phi)$$

      where the parameter λ, which indicates how rapidly the oscillation decays, and the parameter ω', the new oscillation frequency, are given by:

$$\lambda = b/2m; \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\omega^2 - \lambda^2}$$

      where b is the damping constant. So for small damping (low b), the decay is very slow (small λ), and ω' is only slightly less than the undamped angular frequency ω.

(1) The "Underdamped" fit function in Logger Pro is a cosine wave whose amplitude decreases exponentially with time: position = A*exp(-t/B)*cos(C*t+D)+E. Note that the fit parameter B is the inverse of the parameter λ.

(2) The key difference between this and undamped motion is the exponential factor, exp(–t/B). The fit parameter B is called the *time constant* of the motion; it represents how quickly the amplitude decreases. The time constant is usually denoted with the Greek letter tau (τ), but Logger Pro doesn't do Greek letters in curve fitting, so that's why it's B in the equation.

(3) Quantitatively speaking, τ is equal to the amount of time it takes for the amplitude to decrease by a factor of 1/e, where e is the constant 2.718, the base of the natural logarithm. (1/e is about 0.37.)

Here's a graph of exp(–t/τ) for τ=1.00 second:
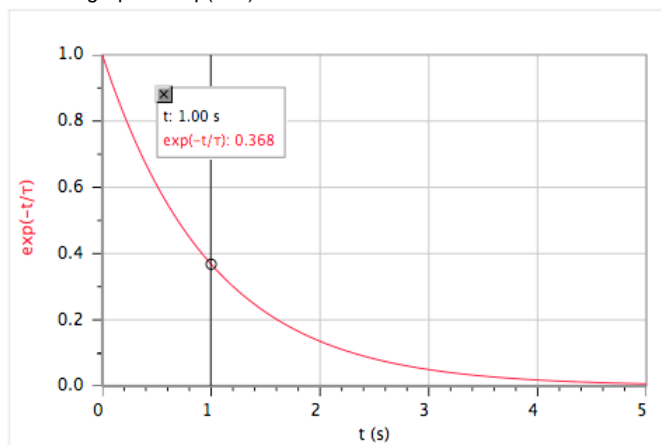
b. If the damping is very large, the system does not oscillate at all. In fact, if it is displaced from equilibrium, it takes a long time for it to even return to its initial position because the drag force is so severe. Such a system is called *overdamped*. It turns out that this behavior is very nearly described by a simple exponential function:

$$x_{overdamped}(t) = Ae^{-\lambda t}$$

where the parameter λ depends on the damping coefficient b in a different way than for underdamped systems. For overdamped systems, λ is always less than ω, the angular frequency of undamped oscillation.

c. The transition from overdamping to underdamping and vice versa is known as *critical damping*. As with overdamping, a critically damped system does not oscillate, but it returns to equilibrium faster than an overdamped system. It also follows (approximately) the negative exponential, but with a larger value of λ, which allows it to return to equilibrium faster than an overdamped system.

In fact, this is the defining characteristic of critically damped systems: **they return to equilibrium quickly and stay there**. An overdamped system is slow to return to equilibrium because it's just slow, period. An underdamped system gets to equilibrium quickly, but overshoots it and keeps oscillating about it, albeit with a gradually diminishing amplitude. This is the reason critical damping is interesting: in many applications (e.g. shock absorbers), you'd like any oscillations to damp out as quickly as possible.

III. Materials
A. 2 long, flimsy springs (type "A")

B. 2 short, stiff springs (type "B")

C. Mass stand and masses

1. The mass stand is a hook with a tray at the bottom for putting masses on it. The stand itself has a mass of 50 grams.

2. There are also masses ranging from 100 g to 500 g in the set.

D. Lab jack

E. Sonar motion detector

1. The sonar motion detector is a sensor that detects the position of objects using sonar ranging:

2. The **minimum distance** away from the sonar detector that objects can be "seen" is 15 cm (about 6 inches). The resolution of the detector is 0.3 mm (that is, an object has to move by at least 0.3 mm in order for the sonar detector to read a different position measurement for it). The sampling frequency of this motion detector is about 30Hz.

F. 600 mL beaker

G. 25 mL graduated cylinder

H. Plastic water bottle

I. Karo corn syrup

J. Stirring rod

K. Forceps

IV. Procedure

A. Before you begin:

1. Take a picture of yourselves using Photo Booth and drag it into the space below:

2. Tell us your names (from left to right in the above photo):

B. Measuring harmonic motion

1. Open the file Lab5.cmbl in Logger Pro.

2. One spring (type A)

a. Using the sonar detector, determine the angular frequency of a 70-g mass (50 g for the stand + 20 g for the CD) oscillating on a type A (long, floppy) spring.

(1) Hang the mass-spring system high over your lab bench and place the sonar detector above it. The CD on the hanging mass is so that the detector can "see" the motion of the hanging mass.
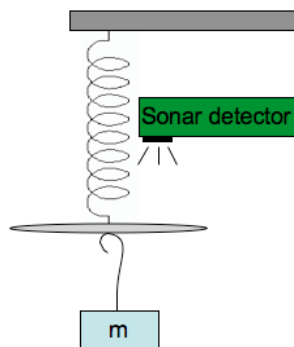
(2) Remember, the sonar detector can only detect objects a *minimum distance of 15 cm away*. Also, use only small amplitudes of oscillation; this will significantly decrease the chances of having the mass fall off the stand and break something.

(3) Pull the mass down and let go. Record data for the motion of the hanging mass for several periods. Try to get everything lined up vertically and minimize side-to-side motion.

(4) After you have taken some data, use it to estimate the period of the oscillation from the graph. Record it here, along with an estimate of your uncertainty:

(5) Try fitting a sinusoidal curve to the position vs time graph. Use the Curve Fit button and under "General Equation," scroll down to the option near the bottom called "Undamped." The general equation for undamped motion is position = A*cos(C*t+D)+E.

Which parameter in the fit corresponds to angular frequency?
What do the other parameters correspond to?

(6) Before clicking "OK", choose "Manual" (rather than "Automatic") from the top right. Manually change the value of the parameter D by clicking the up or down arrow button (and holding it). **What does this do to the fit?**

(7) What is the angular frequency from the fit (don't forget uncertainty)?

ω =

(8) Calculate the angular frequency of the oscillation from the period that you measured in part (4). How does this compare to the angular frequency from the fit (don't forget uncertainties)?

b. Paste a copy of the position vs time graph, including the sinusoidal fit, below:

c. From your data and the sinusoidal fit, calculate the spring constant of the type A spring (don't forget units and uncertainty):
k_A =

d. How does ω change if you have a mass of 270 g, rather than 70 g?

3. Effective spring constants
  a. Now you'll use the same technique to test the predictions you made on the pre-lab for springs in series and parallel configurations.
  b. Measure the spring constant of two type A springs in parallel (ignore uncertainties).
   (1) $k_{effective}$ =

   (2) How does this compare to what you found on the pre-lab?

  c. Measure the spring constant for the type B spring (ignore uncertainties).
   (1) Hint: If the amplitude of the oscillation is small, it will be difficult to fit. How can you increase the amplitude?
   (2) k_B =

  d. Measure the spring constant for two type B springs in series (ignore uncertainties).
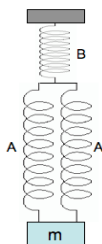   **(1)** $k_{effective}$ =

(2) **How does this compare to what you found on the pre-lab?**

(3) **Does it seem like the two springs stretch by the same amount as the mass oscillates?**

4. Three springs, in series/parallel

a. Connect one B and two A springs in the configuration shown below:

b. Before you actually measure anything, predict what value of $k_{effective}$ you would get from this combination:

$k_{effective}$ **(predicted) =**

c. Measure the spring constant of this configuration

(1) $k_{effective}$ **=**

(2) **How well does this agree with your prediction?**

d. Compare the amount that spring B stretches with the amount that the A springs stretch. Are they equal? If not, which amount is larger?

5. Something to think about

a. Suppose you took a type A spring and cut it in half, to make two shorter springs. **What would the spring constant of one of these springs be?**

$k_{A\ cut\ in\ half}$ **=**

b. If you are curious, we have some half-A springs lying around for you to test your prediction. Ask your TF for help finding them.

C. Exploring damping

1. Damping due to air drag

a. Use m = 150 g and set the oscillation going with an amplitude of a few centimeters. Click Data Collection Data Collection and set it to collect for 2 minutes. Click [▶] Collect and observe the motion of the mass.

(1) **What do you observe?**

(2) **Is the motion underdamped or overdamped?**

b. Fit the data to a curve based on your observation of the motion.

(1) **What happens to tau (or B) as you change the amount of damping?**

(2) Paste a copy of your position vs time graph, including the fit parameters, here:

(3) What is the angular frequency of the oscillatory motion?

ω =

(4) What is the time constant for the motion?

τ =

(5) Looking at the graph, compare the amplitude of the motion at the beginning, and at a time B seconds later. **What factor do they differ by?**

▽**2.** Damping due to viscous drag

a. Pour approximately 300 mL of Karo syrup into the 600 mL beaker and set it on the lab jack. Position the mass over the beaker and raise the height of the jack until the mass is resting in the center of the beaker as shown:
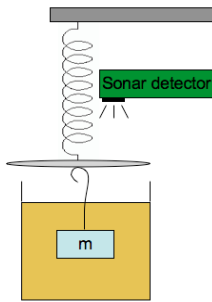
(You should still be using the type A spring with a mass of 150 g.)

▽**b.** Motion of heavily damped systems

(1) With the mass motionless at equilibrium, try to position the collar as level as possible and then zero the motion

detector by clicking [Zero] . It will click a few times and then be still.

(2) Start the data collection by clicking [▶] Collect. Without touching the collar directly, pull the mass up so that the top of the mass is level with the top of the syrup and then release it. After it has come to a complete

stop, you can click on the [■] Stop button to stop the data collection.

(3) What do you observe?

How does this relate to what you know about damped harmonic motion?

(4) Fit the data to a curve based on your observation. Only use the data in the time interval just after the moment you released the mass until the end of the data collection. Check the box marked "Time offset." This will effectively use the moment of release (the beginning of your selected time interval) as t=0, instead of the time you clicked Collect.

(5) When you have a good fit, record the best-fit value of τ (the time constant) here:

τ =

(6) Double-click on the graph and under Graph Options, check the box for "Y Error Bars." Click OK. You should now see tiny error bars take the place of each data point. (You may need to zoom in on the vertical scale in order to see them.) These error bars represent the sensitivity of the motion detector.

(7) From the graph, estimate when the mass reaches a position within one error bar of its final equilibrium position. Subtract the starting time (t when you released the mass) to get the time elapsed until the mass reaches equilibrium:

Time to equilibrium =

(8) Turn to page 2 of the Logger Pro file. In the data table there, enter your time constant in the top row, in the column labeled τ. Enter your time to equilibrium in the last column, labeled "Time to zero." (The third column, ω, doesn't have a defined value because overdamped systems do not oscillate, and hence have no angular frequency.)

(9) *THIS IS AN IMPORTANT STEP, DON'T FORGET TO DO IT!!!*

Turn back to page 1 in the Logger Pro file. Press Apple-L to store this data set. It is now called "Run 1."

c. Now remove the mass from the beaker and take the beaker off the lab jack. **Don't** change the height of the lab jack. Add 30 mL of water to the karo syrup and stir it until it is well-mixed. Because viscous fluids do not mix easily, this will take about 60 seconds of stirring. The effect of adding the water will be to reduce the viscosity and therefore lower the amount of damping in the system.

d. When the water is thoroughly mixed in with the syrup, put the beaker back on the lab jack and the mass back into the beaker. Repeat step b (starting with zeroing the motion detector) for this mixture. Record the time constant and time to zero in the table on page 2 of the Logger Pro file, this time in the row marked "30 mL dilution." ***Be sure to remember to store each data set (Run 1, Run 2, Run 3, etc.) as you go.*** If the graph gets too crowded for you, you can double-click on it and uncheck the boxes for the runs you don't want to show.

e. Add 30 more mL of water, mix well, and repeat. Take data for dilutions of 0 mL, 30 mL, 60 mL, 90 mL, being sure to mix thoroughly for each dilution.

(1) Describe your observations as you dilute the Karo:

(2) Fit the data in each run based off of your observations of the motion. (You should continue to use the Time Offset option, when necessary.) Some things to think about for underdamped systems...

(a) What does tau represent if the system is underdamped?

(b) In calculating the time to zero, should you use the value where the function first crosses zero? Why or why not?

(c) When you fit the equation for underdamped motion, you can also determine the angular frequency of the oscillation, ω. Enter a value for ω into the data table on page 2 of the Logger Pro file for every dilution which falls into the underdamped regime.

f. After you have completed all of the dilutions up to 90 mL, go to your data on page 2 and see which one came closest to **critical damping** (the threshold between over- and underdamped motion).

(1) Which dilution was it?

(2) What was the time constant when the system was closest to critical damping? the time to equilibrium?

τ =

Time to zero =

(3) Paste a graph of position vs time for the system when it was closest to critical damping:

(4) Create a graph showing both τ and Time to zero vs dilution and paste it here:

In words, what conclusions can you draw from this graph regarding the relationship among τ, Time to zero, and the regimes of damping?
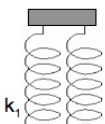
(5) Look at the values of ω that you observed for the underdamped systems. **How do they compare to the ω you got for the same system damped only by air drag? For underdamped systems, does ω increase, decrease, or stay the same as the amount of damping goes up?**

V. Conclusion

A. When you have finished, <u>clean up anything in your work area that has has karo syrup all over it</u> (the beaker, mass stand, masses, stirring rod, forceps if you used them, and anything else which was in the splash radius of your karo syrup) <u>by taking it to the sink and rinsing it out thoroughly with warm water. Leave the glassware to dry on the tray **at your station**</u> (not at the sink).

B. Submit your lab report online on the course's website on Sapling Learning. If you don't remember your password, have problems signing in, or the system isn't working, tell your TF.

C. Maybe this week, you could gather up all of your things and take them with you when you leave the lab. You know, calculators, notebooks, phones, iPods, lab handouts, even pencils & pens. It's just a thought.

VI. Pre-Lab Assignment

These questions are here for reference.

A. Suppose you connect two springs k_1 and k_2 to the same mass m in *parallel*, as in the diagram at left. Assume the springs have the same equilibrium length. What is the effective spring constant of this two-spring setup, i.e., what is the proportionality constant between the total spring force on the mass and its displacement from equilibrium?

B. Now consider the arrangement at right, where the two springs are connected in *series*. What is the effective spring constant for this configuration? (*Hint*: you'll need to draw more than 1 FBD.)