Lab 6: Fluids and Drag

I. Introduction

A. Learning objectives for this lab:

1. Learn how fluids such as blood exert a pressure that varies with height

2. Understand the motion of objects in fluids and how it depends on viscosity and density

B. Review the second half of Lecture 7 and pages 121-122 of Bauer & Westfall on drag forces. It's been a while since we dealt with these concepts, so you'll want to have them fresh in your mind for this lab.

C. In this lab, you'll explore the physics of fluids, both through static properties (*e.g.,* pressure and buoyancy) and phenomena related to fluid flow (*e.g.,* viscosity and drag). You'll measure your own blood pressure and . You'll also explore drag forces on a falling sphere in a viscous liquid and use the concept of terminal velocity to characterize the fluid's viscosity and density. Finally, you'll learn how to estimate Reynolds numbers and how they can be used for modeling purposes.

II. Background

A. Static pressure and buoyancy

1. In a static column of fluid, the pressure in the fluid increases with increasing depth. For a fluid of density ρ in a column of height h, the pressure difference between the top and bottom of the column is

$$P_{\text{bottom}} - P_{\text{top}} = \rho g h$$

2. Due to this pressure difference, an object submerged in a fluid appears to weigh less than it would in a vacuum, because the fluid pressure pushing up on the bottom of the object exceeds the pressure pushing down on the top. This difference results in a net upward force called the **buoyant force**. Archimedes's principle states that *the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object*. There are two important cases to consider:

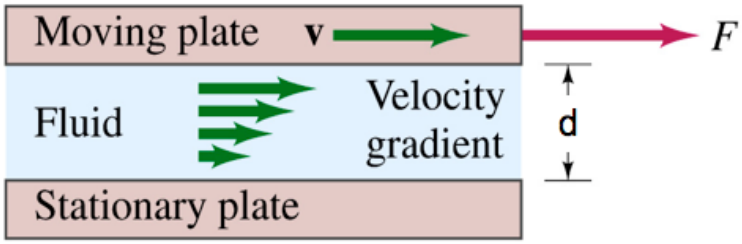
a. For an object that is *completely submerged* in the fluid, it displaces an amount of fluid equal to its own volume. Therefore, the buoyant force is equal to the density of the fluid multiplied by the volume of the object multiplied by g. This is true whether the object is static or moving in the fluid.

b. For an object that is *partially submerged* in the fluid, the volume of fluid displaced is only a fraction of the object's total volume. However, if the object is known to be in static equilibrium and there are no forces acting on it (other than buoyancy and gravity), then the buoyant force must be exactly equal and opposite to the weight of the object. Thus *a static floating object displaces an amount of fluid equivalent to its own weight*.

B. Viscosity

1. Viscosity is a property of a fluid that opposes relative motion. You can think of viscosity as being due to the frictional force between adjacent layers of fluid as they slide past each other.

2. For a more technical definition, consider the following situation: two parallel plates of area A are separated by a fluid of thickness d:



You now want to move the top plate while keeping the bottom plate fixed. Due to viscosity, you need to apply a force F to the top plate in order to move it at a constant speed v. The layers of fluid between the plates will have different velocities: the layer at the very bottom doesn't move at all, and the layer at the top moves at the same speed v as the moving plate with a velocity gradient in between.
For many fluids, over a wide range of temperatures and pressures, the amount of force F that you need to apply is proportional to the speed v at which you want to move the plate and to the area A of the plates and inversely proportional to the distance d between the plates. The viscosity of the fluid is then *defined* to be the proportionality constant:

$$\eta = \frac{Fd}{vA}$$

where viscosity is represented by the Greek letter eta (η). Fluids that obey this simple proportionality are called *Newtonian* fluids.

3. Viscosity has dimensions of [M] / [L][T]. The SI unit of viscosity is the $N \cdot s/m^2$ or $Pa \cdot s$ (Pascal-second). At room temperature, the viscosity of water is about 10^{-3} $Pa \cdot s$. (However, this number can change by a factor of two with only a few degrees' difference in temperature.) The viscosity of air is about 2×10^{-5} $Pa \cdot s$, although it, too, depends on the temperature.

C. Drag forces and terminal velocity

1. Objects that are moving in a fluid medium experience **drag forces** which oppose their motion, much like friction. Unlike our model of friction, however, the magnitude of the drag forces is *velocity-dependent*: the drag increases as the object's speed relative to the fluid increases. (Kinetic friction, by contrast, is generally taken to be a constant if the normal force is also constant.)

2. There are two kinds of drag forces:

a. Pressure drag is due to the fact that the fluid has mass, and in order to move through the fluid, you have to push the fluid out of your path. The magnitude of the pressure drag on an object of cross-sectional area A moving at speed v through a fluid of density ρ is:

$$F_{\text{pressure drag}} = \tfrac{1}{2} C_d \rho A v^2$$

where C_d is a dimensionless constant called the *drag coefficient*, which can depend on the shape of the object and roughness of its surface. Typical values of C_d are about 0.1 to 1.

b. Viscous drag is due to the fact that fluids have viscosity, which opposes shearing of the fluid. For a solid sphere of radius r moving at speed v in a fluid of viscosity η, the viscous drag is given by the Stokes formula:

$$F_{\text{viscous drag (sphere)}} = 6\pi\eta r v$$

For shapes other than a sphere, the exact formula varies, but in all cases the viscous drag is directly proportional to speed, viscosity, and the linear size of the object.

3. For most situations, one kind of drag force is much greater than the other, so the smaller one makes a negligible contribution to the total drag on the object.

- **a.** For small objects which are moving slowly, viscous drag dominates.
- **b.** For large objects which are moving quickly, pressure drag dominates.
- **c.** This raises the question of what size is considered to be "small" and what speed is considered "slow." The answer depends on the viscosity and density of the fluid. One way of looking at this is to consider the ratio of pressure drag to viscous drag. Since the area of an object is proportional to the square of its linear size l, neglecting factors like 1/2 and π we get (roughly):

$$\frac{F_{\text{pressure drag}}}{F_{\text{viscous drag}}} \approx \frac{\rho \ell^2 v^2}{\eta \ell v} = \frac{\rho \ell v}{\eta} = \text{Re}$$

The fraction $\rho l v/\eta$ is called the **Reynolds number**, abbreviated Re. Because we got it by dividing one force by another force, the Reynolds number has no dimensions or units. When Re is much smaller than 1, viscous drag dominates; if Re is much greater than 1, pressure drag dominates.

- **(1)** We'll see Reynolds number in lecture later in the semester. It turns out to be a very useful quantity for characterizing fluid flow.
- **(2)** One important thing to note is that the same fluid (i.e. the same ρ and η) can give you vastly different Reynolds numbers depending on the size and speed of the flow (l and v). For example, an aircraft carrier moving through water has a Re of about 10^9; a swimming goldfish might have a Re of about 10^2; and a bacterium in the same water might have a Re of only 10^{-5}.
- **(3)** Conversely, if two flows have the same Re, then the physics in each is essentially the same regardless of the size, speed, or fluid involved. For example, a bacterium in water (with Re on the order of 10^{-5}) and a millimeter-sized bead in honey (Re also about 10^{-5}) behave very similarly. But you cannot model a bacterium in water by a macroscopic object moving in water at macroscopic speeds because the Reynolds number would be totally different. One is dominated by viscous drag, and the other is dominated by pressure drag.

4. Consider an object under the influence of a constant external force (e.g. gravity) that is also subject to drag, either pressure drag or viscous drag.

- **a.** If the object is initially at rest, the drag force is zero, so there will be no force to oppose gravity and the object will accelerate downwards.
- **b.** However, as it accelerates, the drag force increases to oppose the downward motion, and the faster it goes, the larger the drag force gets. Eventually, the object will be moving fast enough that the drag force is large enough to *exactly cancel the external force*.
- **c.** When this occurs, the net force on the object becomes zero, and it stops accelerating, which means its velocity remains constant. This final velocity is called the **terminal velocity**.
- **d.** Terminal velocity is extremely useful, because we know that *when terminal velocity is reached, the sum of the forces is zero.* That means, in this case, that the drag force exactly balances the external force. This fact enables us to explore the drag force. If we can measure the terminal velocity for several different values of the external force, we can determine how the drag force depends on velocity. This technique is much easier than attempting to vary v and measure the drag force directly.

III. Materials

A. Digital video camera

B. 600 mL beaker filled with yummy karo syrup

C. Forceps

D. Box containing spheres of different materials, all with 1/16" radius

E. Blood pressure sensor

1. The blood pressure sensor consists of an inflatable cuff which connects to a pressure sensor, which is the small box that connects to the computer and interfaces with Logger Pro.

2. Blood pressure is typically measured with two numbers: the **systolic** and **diastolic** pressures. This is because your blood is not a static fluid: the pressure in it varies during each pulse beat as blood is pumped through the body. Both systolic and diastolic pressure are reported in units of mmHg (millimeters of mercury). A blood pressure of "120 over 80" means a systolic pressure of 120 mmHg and a diastolic pressure of 80 mmHg.

a. The systolic pressure is the maximum pressure during a pulse, which occurs near the beginning of a cardiac cycle.

b. The diastolic pressure is the minimum pressure during a pulse, which occurs during the resting phase of a cycle.

3. The cuff is inflated by pumping on the bulb end of the tube. Next to the bulb is a valve; pressing this valve releases the air from the cuff. The valve also contains a screw which can be turned to fine-tune the rate at which air leaks from the cuff. Turning the screw clockwise increases the leak rate; turning it counterclockwise decreases the leak rate.

4. The blood pressure sensor works by inflating the cuff to a high enough pressure to actually cut off blood flow in your brachial artery (inside your arm). The pressure transducer then measures the pressure inside the cuff as a function of time as the cuff gradually deflates by leaking air. As you can see from your upper graph (Cuff Pressure vs. Time) if you zoom in or from the lower graph (Oscillatory Amplitude vs. Time), there are small "blips" in the cuff pressure every time your heart beats. This is due to your heart trying to pump blood through the blocked artery. Since blood can't get through, there is a temporary accumulation of blood in your artery. Your artery slightly expands in volume because the blood is incompressible. This expansion occurs at the expense of the volume of the cuff, leading to a small increase in air pressure inside the cuff. When the heartbeat subsides, the pressure returns to its previous level.

IV. Procedure

A. Before you begin...

1. Take a picture of yourselves using Photo Booth and drag it into the space below:

2. Tell us your names (from left to right in the above photo):

B. Blood pressure

In this part, you will each measure your own blood pressure at your upper arm using the blood pressure sensor and think a little bit about how the sensor works.

1. **<u>NOTE:</u> If you do not feel comfortable performing any portion of the lab, feel free to borrow a friend's data for the portion in question.**

2. Start Logger Pro (*without* opening a particular file). After a few seconds, it should detect the blood pressure sensor and open a page with custom blood pressure readings.

3. **<u>Important safety warning:</u>** Before you begin taking data, make sure that you know how to release the pressure in the cuff (by pressing on the valve). If the pressure in the cuff gets high enough to be painful to the patient, release it **immediately**. For most people, it will not be painful to reach a pressure of 170 or 180 mmHg (though it is mildly uncomfortable; after all, the idea is to cut off blood flow to their arm temporarily).

4. Blood pressure and volume changes

a. Have the "patient" sit upright in a chair and, if possible, remove outer layers of clothing and roll up the sleeve as far as possible. (If the sleeve is too tight to roll up, it is also fine to place the cuff over one thin layer of clothing.) Wrap the inflatable cuff around the patient's upper arm so that the prickly velcro surface (and the label "INDEX ➡") face **outward**. Also, turn the cuff so that the two rubber

hoses are on the **inside** of the patient's arm by the bicep. The bottom of the cuff should be about 2 cm above the elbow joint.

b. When the patient is ready, click on the  button in Logger Pro to begin data collection. **The patient must keep her arm and upper body completely still throughout the measurement.** Rapidly inflate the cuff (using full pumps rather than small quick pumps) until the pressure reaches 160 and then wait. You will see the pressure slowly decrease in the upper graph; the patient will feel (and maybe even hear) the pulses in her arm. After about 40 seconds, the oscillatory "peaks" will appear in the lower graph. These peaks are used by the software to calculate systolic and diastolic pressures.

c. Eventually the lower graph will stop updating itself; at this point the data collection is complete. You can read off the systolic and diastolic pressures from the meters on the screen.

d. If there is no reading after 120 seconds, or a clearly meaningless result (e.g. systolic less than diastolic), you can try again. One common problem is that the cuff pressure should leak at a rate between 2 and 4 mmHg per second. If it is leaking too slow or too fast, you might not get a reading. You can adjust the leak rate using the screw on the release valve.

e. The lower graph (Oscillatory Amplitude vs. Time) shows the cuff pressure, except it subtracts off the overall decreasing trend of the cuff pressure as the air slowly leaks out of the valve.

f. Patient #1

(1) What are your systolic and diastolic blood pressures (in mmHg)?

(2) Estimate the volume of air in the cuff when it is wrapped around your arm and inflated. The cuff is 14 cm wide. *Hint*: you will have to estimate your arm's radius (no pun intended) and how thick the cuff is when it is full inflated.

(3) The air in the cuff obeys Boyle's Law (PV = constant) during the time when it is wrapped around your arm. Assume that (the volume of your arm) + (the volume of the cuff) is a constant, such that the increase in volume of your brachial artery during each pulse is equal to the amount by which volume of air in the cuff decreases. **Using your data, estimate by how much the volume of your arm changes during each heartbeat.** *Hint*: if the changes in P and V are small, the equation PV = constant can be written as $\frac{\Delta P}{P} = -\frac{\Delta V}{V}$.

5. Height dependence on blood pressure

a. Attach the blood pressure cuff to the next patient's ("Patient #2") arm.

b. What are Patient #2's systolic and diastolic blood pressures (in mmHg)?

c. Retake Patient #2's blood pressure with his/her arm raised above his/her head. **What are your systolic and diastolic blood pressures in this case? By how much do these values differ compared to the previous values? Why do they differ?**

d. Based on the height difference, calculate how much the blood pressure in your arm should differ by in the two configurations. How does this compare to your measurement?

C. Terminal velocity

In this part of the lab, you will drop small spheres into a viscous fluid (karo syrup, which will be familiar to all of you from the previous lab) and use the terminal velocity to determine the viscous drag on each sphere.

1. Open the file Lab6.cmbl in Logger Pro. Logger Pro will prompt you on whether you want to set up sensors. Click the button for Use File As Is.

2. Set up the file to take video captures:

a. Go to the Insert menu in Logger Pro and select "Video Capture..."

b. Select "DV Video."

c. Select the default values for the resolution and the sound source.

d. Click the Options button in the Video Capture window and set the following options:

(1) Video Capture Only

(2) Capture Duration: 20 seconds

(3) Capture File Name Starts With: Teflon

(4) Click OK.

3. Position the camera so that you can see the beaker of karo syrup. Make sure you have something in frame in order to set the scale of your video.

4. Using a pair of forceps, pick the white **teflon** sphere out of your box of different spheres and hold it in the karo syrup about an inch below the surface. Release the sphere and then remove the forceps from the fluid.

5. Using video analysis, determine the terminal speed with uncertainty of the falling sphere. Enter these values in the data table on page 5 of the Logger Pro file.

6. Paste your graph of the sphere's *y*-position vs time below. Is there a time when the sphere is accelerating? If so, when? If not, how do you know?

7. Go back to page 2 of the Logger Pro file and repeat the procedure for each ball. Each sphere has its own page in the Logger Pro file for you to use. There are several key differences to note each time:

- **a.** When setting the Video Analysis Options, change "Capture File Name Starts With" to the name of the material for each sphere that you drop.
- **b.** Likewise, when setting the Data Set Options after you do each analysis, change the data set name to the name of the material. That makes it much easier to keep track of which data set corresponds to which sphere.
- **c.** You don't have to paste the graph and answer questions about it for each ball. Just record the terminal velocity and its uncertainty in the data table on page 5 for each sphere that you drop.

8. Analysis of the data

- **a.** Now go to page 5 of the Logger Pro file. You should see a data table populated with the density of each material (which is given), as well as terminal velocities (along with uncertainties) of each sphere. **Create a graph of density vs. the terminal velocity. Paste a copy of it here:**

- **b. How can you determine which type of drag force dominates (pressure or viscous) from a plot of density vs. terminal velocity?**

- **c.** In the prelab you assumed viscous drag was the dominant drag force. **Does the shape of your graph support the Stokes equation? How did you come to this conclusion?**

- **d.** Fit a line to your data and determine the following parameters:
  - **(1) Best-fit slope with uncertainty =**
  - **(2) Best-fit intercept with uncertainty =**
- **e. From your data, what is the density of karo syrup with uncertainty?**

  - **(1) How does this compare with other known densities (*e.g.*, air, water, the materials of the spheres)?**

  - **(2)** By pouring karo syrup into a graduated cylinder on a precise balance, we directly measured its density to be 1.36 ± 0.01 grams per mL. **Does your measurement agree with this value?** (Hint: How does one answer scientific claims such as this?)

- **f. Using your data, calculate the viscosity of karo syrup with uncertainty.** The manufacturer reports that the **diameter** of the spheres is 125 ± 2 mils (1 mil is a thousandth of an inch).

  - **(1) How does this compare with other known viscosities?** (The viscosity of water is about 10^{-3} Pa · s, canola oil is about 10^{-1} Pa · s, motor oil is 1 Pa · s, honey is 10 Pa · s, and various types of lava have viscosities of 10^{2} Pa · s and up.)

- **g.** Nylon has a density of 1060 kg/m^3. **From your fit, <u>predict</u> the terminal velocity of a nylon sphere of the same size (1/8" diameter) submerged in karo syrup.**

  Does your answer make sense? Perform the experiment to convince yourself.

- **h. Based on your calculated density and viscosity, estimate the *largest* Reynolds number of the different spheres dropping. Is it still small enough that viscous drag is a good**

assumption?

V. Conclusion

A. When you have finished, cover your beaker of karo syrup with plastic wrap. Take your forceps and anything else that may have been splattered with syrup and ***wash them off at the sink in the back of the room***.

B. Submit your lab report online. Remember to save your work as a pdf file before uploading it to sapling.

C. Don't forget to take all of your belongings with you when you leave the lab.

VI. Pre-Lab Questions

This question is here for reference.

A uniform sphere of radius r and density ρ is submerged in a fluid whose density is ρ_{fluid}. The object begins to sink under the influence of gravity. Assume that the magnitude of the drag force is given by Stokes's Law: $F_d = 6\pi\eta rv$, where η is the viscosity of the fluid and v is its speed.

A. Write down Newton's 2nd Law (in the vertical direction) for the object and calculate the terminal speed $v_{terminal}$ of the sphere in terms of ρ, ρ_{fluid}, r, g, and η.

B. Suppose you conduct this experiment several times for spheres of the same radius but different densities ρ, all sinking in the same fluid. (The density of each sphere is known, as is the common radius.) Each time, you measure the terminal speed $v_{terminal}$ of the sphere. You then construct a plot of ρ versus $v_{terminal}$. Derive an equation relating ρ to $v_{terminal}$ showing that the graph would be a straight line. If you knew neither the viscosity nor the density of the fluid, how could you determine them from a line of best fit?

C. Suppose that instead of obeying Stokes's Law, the drag force were instead proportional to v^2 (rather than to v). How would this affect the shape of the ρ vs $v_{terminal}$ graph? Would it still be linear?