Background and Introduction

Rounding and significant figures

A little boy and his mother walk into the natural history museum and see a huge dinosaur skeleton in the lobby. "Wow!" says the boy. "I wonder how old those bones are?"

A nearby security guard answers, "That skeleton is 90,000,006 years old!"

"90 million *and six*? Are you sure?" asks the boy's mother.

"'Course I am," replies the security guard. "I asked the curator how old it was when I started working here, and he said 90 million years. And I've been working here for six years."

Okay, it's a silly joke, but it gives us pause for thought. What's wrong with the security guard's answer? The uncertainty in the age of the skeleton is on the order of millions or tens of millions of years. So it's preposterous to make any claims as to the ones digit; the six years don't matter in the least. Even if we don't necessarily quote uncertainties when tossing numbers around in everyday conversation, it's still absurd to make such a huge error in the number of significant figures.

Arguably, it's even more absurd to make such an error when you are, in fact, quoting the uncertainty along with the value. For example, on lab reports in the past, students have made claims such as:

"We measured a value of 1.4969461 ± 0.27777777."

If the uncertainty in the value was about 0.28, why would you claim to know the result to seven decimal places? Just because that's what your calculator said when you plugged in the numbers? Don't believe everything that your calculator tells you! Keep in mind what the numbers mean—and what they don't mean. In this case, no matter what your calculator says, you know full well that you have zero knowledge of even the hundredths digit of the answer, and only partial knowledge of the tenths digit.

Here's a rule of thumb you can rely on: round the uncertainty to one significant figure. Then round the answer to match the decimal place of the uncertainty. In our example above, we would just say that the uncertainty is 0.3, and the actual value would be rounded to the nearest tenth in accordance with the uncertainty. So the final result would be:

"We measured a value of 1.5 ± 0.3."

If the uncertainty were only 0.0027777, we'd call it 0.003 and the result would be 1.497 ± 0.003. The small uncertainty gives us confidence that the thousandths digit is meaningful.

One exception to the rule of thumb: If rounding the uncertainty to one significant figure would cause that figure to be a 1, then you keep the next digit as well. So for instance, if you have 5.83333333 ± 0.14285714, then you would round the uncertainty to 0.14 (instead of just 0.1) and report 5.83 ± 0.14 (instead of 5.8 ± 0.1).

Momentum

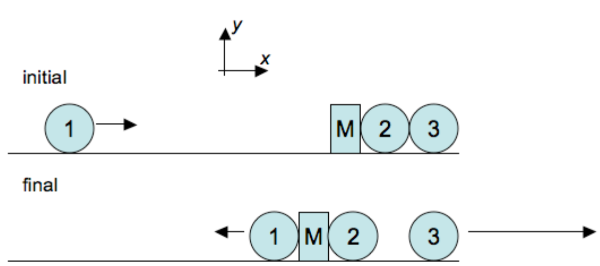
You've already learned about momentum in the first few weeks of lecture, section, and homework. But here is a summary of the big ideas:

- The total momentum of an isolated system is conserved.
- The momentum of a single object is its mass times its velocity. Momentum is a vector.
- The total momentum of a system is the vector sum of the momenta of each object in the system.
- An isolated system is one that has no interactions with anything outside of the system.
- A system can be considered to be functionally isolated if there is no net external force on the system.
- The impulse on a system is defined as the *change* in the system's total momentum between some initial state and some final state (final momentum minus initial momentum).
    - For an isolated system, therefore, the impulse is zero no matter what the initial and final states are chosen to be.
    - For a non-isolated system, the impulse on the system during some interaction is given by the average external force on the system multiplied by the time duration of the interaction.

The Gauss Gun

In this lab you will explore the conservation of momentum in an interesting physical system: the Gauss gun. As you saw in lecture and in the online videos, the Gauss gun can be used to shoot steel ball bearings at high speeds. The details are complicated because they involve magnetic forces, which we won't cover in this course. However, the beauty of momentum conservation is that we don't have to know or care about the details of the interaction: we just look at the initial momentum and the final momentum and compare them. You will do an experiment to answer the question: "Is momentum conserved during the firing of the Gauss gun?"

Let's start by defining the system:

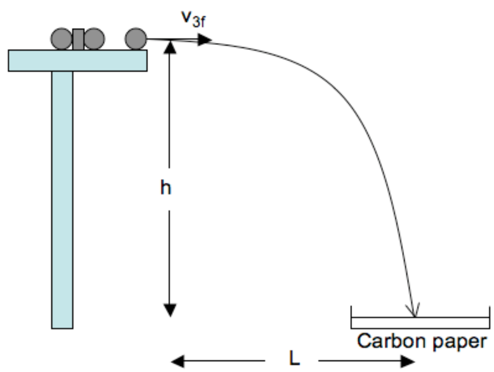


- There are three balls (1, 2, and 3) and a magnet (M). Initially, ball 1 rolls to the right (in the +*x* direction), while the M-2-3 trio are stationary.
- Then the big magnetic interaction occurs. However, if the system is defined as 1+2+3+M, then that big magnetic interaction is an *internal interaction*, so it does not change the total momentum of the system.
- In the final state, ball 3 shoots off to the right very fast, while the 1-M-2 trio recoil to the left somewhat slowly.
- We'll be analyzing the interaction using the video camera, which is limited to 30

frames per second. It is likely that we will not be able to capture the exact instant of the collision in the video, because it will occur between frames. So let's make a specific definition: for the purposes of this

  - "initial" refers to the *last video frame before the magnetic interaction between ball 1 and the magnet*
  - "final" refers to the *first video frame after the collision*

- In order to talk about momentum, we'll have to measure some masses (easy), and three different velocities (harder): the initial velocity of ball 1, the final recoil velocity of 1-M-2, and the final velocity of ball 3. The first two will be calculated from the video. However, ball 3 shoots out too fast to capture in the video. Instead, we'll figure out how fast it is launched using the following setup:

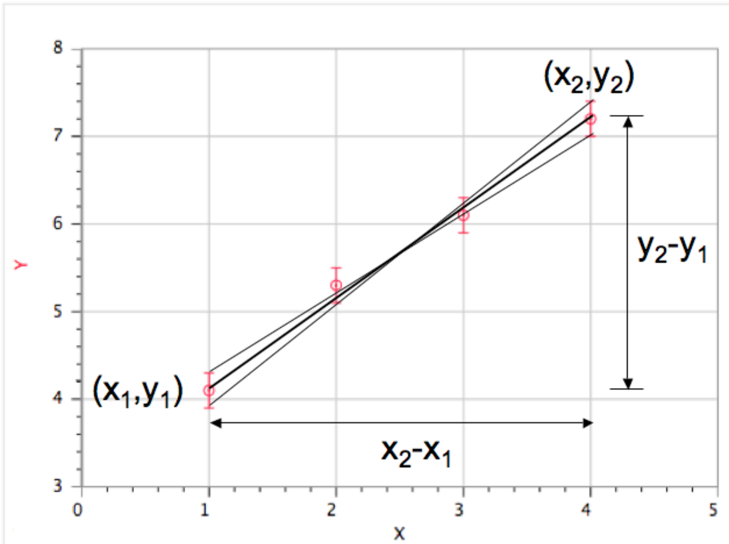


- The track is positioned so that it ends right at the edge of the lab bench. Ball 3 will be launched off the end of the track, and it will strike a piece of carbon paper on the floor, leaving a dark mark. If we measure the height of the bench h and the horizontal distance L from the end of the track to the mark on the paper, we can calculate the velocity with which the ball was launched (assuming it is initially moving in a purely horizontal direction).

<u>Calculating the uncertainty of a slope</u>

As with individual measurements, we can *approximate* the uncertainty associated with linear fit parameters. A simple way of doing this is to draw the steepest and shallowest lines that are consistent with the error bars on the points, and calculate the slope for each.

If there are vertical error bars the three lines (best fit, steepest, and shallowest) might look like this:



We can approximate the uncertainty in the slope as:

$$\delta m \approx \frac{m_{\max} - m_{\min}}{2}$$

Here δm is the uncertainty in the slope, $m_{\max}$ is the slope of the steepest line (found by using the lowest possible value for the first point and the highest possible value for the last point), and $m_{\min}$ is the slope of the shallowest line (found by using the highest possible value for the first point and the lowest possible value for the last point).