Background and Introduction

In this lab, you'll explore Newtonian mechanics of an extended, non-rigid object: your own body. Whenever your center of mass accelerates, it must be due to a net *external* force; conversely, any time there is a next external force on your body, it will result in motion of your center of mass. This motion can be extremely non-obvious, because your center of mass can move as a result of a shift in body position rather than an overall movement of your entire body.

Error propagation: Advanced techniques

The handout on error propagation (included in the back of the green binders) introduces new techniques and equations for doing error propagation problems. By understanding and using these techniques, you can save yourself quite a bit of calculation. The uncertainties calculated using these techniques are also generally a bit more realistic and a bit smaller than those calculated using our old "worst-case" method.

Recall that our existing method is to "crunch three times": once to get the mean (middle) value of the final answer, once to get the largest possible value, and once to get the smallest possible value. Then the mean gives the value, and (largest–smallest)/2 is the uncertainty. By using the new techniques instead, you still have to crunch once to get the value (mean), but now you only have to do one more calculation to get the uncertainty. Sometimes the expression for the uncertainty can look complicated or daunting, but there are useful approximations that we can use, relying on the fact that we only really need to know the uncertainty to about one sig fig.

A word of warning: many of the new error propagation formulas are most easily used and remembered if they are stated in terms of *fractional uncertainty* (or percent uncertainty), instead of absolute uncertainty. Make sure you are comfortable with both and can recognize the difference. As an example, if I measure a length to be 2.00 ± 0.12 m, the uncertainty in the length is 0.12 m (or 12 cm), but I could also say that the length is 6% uncertain (with no units), because 0.12 m divided by the value of 2.00 m is 0.06.

Error propagation should be done using the new techniques if at all possible. However, **the worst-case method will be good enough for PS2** applications. There are two main ways in which the new techniques might be inapplicable:

a. When your calculation involves *correlated errors*. It is often, but not always, possible to re-write your equation so that the errors become uncorrelated. If you can't, you'll have to fall back on the worst-case method.
b. When your calculation involves some mathematical function other than add/subtract/multiply/divide/raise to a power, e.g. sin or exp or log. For instance, if you measure an angle to be 30° ± 2°, what's the uncertainty in the sine? Again, you can fall back on the worst-case method: (sin 32° – sin 28°)/2.

If you are curious about the derivations of these equations from Gaussian statistics, you can look them up in chapter 5 of Taylor.

Newtonian mechanics of an extended object

Recall Newton's 2nd Law for a particle or simple object: the sum of the forces on the object is equal to its mass multiplied by its acceleration. At this point in the course, you've used this fact many times to solve physics problems. Typically, we proceed by drawing the free-body diagram of an object and adding up all the forces on it and then using Newton's 2nd Law to equate the total force to the object's mass times its acceleration.

But what about an extended object or system, which can have many moving parts? Which acceleration are we talking about? Remember that we defined the concept of *center of mass*. For an extended object the sum of all the forces on the system is equal to just the sum of the *external* forces on the system. (The external forces all have 3rd-Law partners, but they are not forces that act on the system, so we do not consider them when using the 2nd Law.)

We can also write Newton's 2nd Law in terms of the rate of change the total momentum of the system's center of mass. If the total mass of the system is constant, this simplifies to the total mass multiplied by the rate of change of the velocity of the center of mass.

The upshot of all of this is: the sum of all external forces on a system is equal to the total mass of the system multiplied by the acceleration of the system's center of mass:

$$\sum \vec{F}_{\text{external}} = M_{\text{total}} \vec{a}_{\text{CM}}$$

We'll use this relation to study the mechanics of the human body, which will first require us to think about where the center of mass of a person is.

Jumping

Whether you are standing or jumping, the only external forces acting on your body are the normal force from the ground and gravity. How can you jump? Raising your center of mass into the air seems like it would require gravity to diminish or the normal force to grow. Why would either of these happen?

The one thing we know for sure is that gravity isn't changing. Therefore, it must be that the normal force from the ground increases as you jump, propelling you into the air. Because you push off harder on the ground with your legs, by Newton's 3rd Law, the ground must also be pressing harder on you.

Before collecting jumping data using video and force plate at the same time

- Set up Logger Pro to take video captures. Here are the Video Capture Options... we'll need:
    1. Video Capture Mode: Video Capture Synchronized with Data Collection
    2. Duration: 10 seconds
    3. Make sure "Time-Lapse Capture" is **not** checked.
    4. Capture File Name Starts With: MovieCapture
    5. Click on the Camera Settings button. In the Compression tab, under Motion, Frames per Second, insert 30, then click OK. (Don't worry if it tells you that the compression mode is not the default.) Click OK again.
- Zero the force plate
- Have one of your group members stand on the force plate facing the camera. Make sure their center of mass label is clearly visible. Another group member should stand next to them holding a meter stick. You will use the whole meter stick to set the distance calibration for the video. Make sure you can see the person's feet on the force plate.

Synchronize the movie to the force data

You will use the moment your feet lose contact with the force plate to synchronize the video to the force data. (Although both are collected at the same time, they are on slightly different clocks so they are not really synchronized to begin with.)

In the lower-right corner of the movie window, click the  button.

1. A synchronization dialog box will open. First, click in the graph on the moment of takeoff. You should see the time corresponding to your click appear in the dialog box.
2. Next, play through the video frame by frame until you find the very first frame where your feet leave the force plate. The time corresponding to this frame will also appear in the synchronization dialog box.
3. Click OK in the dialog box to sync the movie

Once the movie is synchronized with the force data, you can use the "Examine" button in the graph and the movie will automatically show the frame corresponding to the time you are examining. You can also set up a replay control (Analyze → Replay...) to watch the force plate trace and the movie at the same time, at whatever speed you like.