Background and introduction

In this lab you will measure your moment of inertia, as well as explore conservation of angular momentum and precession. Rotational motion is different from linear motion in ways that can be confusing and counter-intuitive. The exercises and questions in this lab will help you get a feel for rotational motion and angular momentum.

Conservation of energy

The work-kinetic energy theorem states that the net work done by the forces acting on an object is equal to the change in that object's kinetic energy. Kinetic energy is the energy of motion, and an object can have translational motion (motion of the center of mass) as well as rotational motion. There is a kinetic energy associated with this rotational motion, and it can be calculated using the following formula:

$$K_{rot} = \tfrac{1}{2} I\omega^2$$

where I is the moment of inertia of the object, and ω is the angular velocity.

In the absence of work done by non-conservative forces, the mechanical energy of a system, that is, the sum of the total kinetic energy and total potential energy, will be conserved. This is useful when analyzing the behavior of the system; no matter how the kinetic and potential energies change, the sum of kinetic plus potential will always be the same.

Moment of inertia

The rotational analogue of mass is called moment of inertia, or rotational inertia. It characterizes the motion of a rigid object that rotates. The moment of inertia of a very small object that has a mass *m* and rotates around an axis a distance *r* away is given by $I = mr^2$. The moment of inertia of a large object rotating around a given axis can be found by imagining the object being composed of small pieces, finding the contribution that each piece makes to the moment of inertia, and then adding up the separate contributions:

$$I = \Sigma m_i r_i$$

The moment of inertia of rigid bodies depends not only on the mass of the object, but on how that mass is distributed and the location of the axis of rotation. The formulae for calculating moment of inertia for various objects with uniform mass density can be found on the Internet and in textbooks.

Torque and angular momentum

When a force is applied on an extended object, three things can happen: the object remains static, the object moves, or the object deforms. One type of motion is rotational motion. The way a force applied to an object results in rotational motion is through a quantity called torque. Torque is the rotational analogue of force, and it depends not just on the force applied ($\vec{F}$), but also on the displacement from the axis of rotation to the location of application of that force ($\vec{R}$):

$$\vec{\tau} = \vec{R} \times \vec{F}$$

The magnitude of torque can be calculated thusly: $\tau = RF\sin\theta$, where θ is the angle between the vectors $\vec{R}$ and $\vec{F}$. Since torque is a vector, it also has a direction. The direction of the torque vector is determined by the right-hand rule, shown by the handy (!) summary below.

HAND – make sure you use your RIGHT hand
HAND – point your right hand in the direction of $\vec{R}$
FINGERS – rotate your arm such that you can bend or curl your fingers so that they point in the direction of $\vec{F}$
THUMB – stick your thumb out; it points in the direction of the torque

Note that the torque vector is perpendicular to both the force vector and the displacement vector.

Just as we have Newton's Second Law for linear motion, relating the net force on an object with the rate of change of the object's linear momentum with time:

$$\Sigma\vec{F} = \tfrac{d\vec{p}}{dt}$$

we have Newton's Second Law for rotational motion, relating the net torque on an object with the rate of change of the object's angular momentum with time:

$$\Sigma\vec{\tau} = \tfrac{d\vec{L}}{dt}$$

The angular momentum of an object depends on the object's moment of inertia, as well as the object's rotational or angular velocity:

$$\vec{L} = I\vec{\omega}$$

Some of the strangeness of rotational motion arises from the fact that an object's moment of inertia can change, not just it's angular velocity, whereas we never encounter situations where an object's mass changes (unless we are dealing with rocket science). The fact that torque is perpendicular to force also leads to counter-intuitive behavior, like precession, explained below.

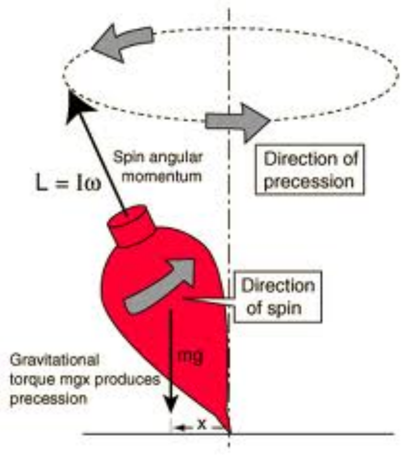
Precession

Motion in which a torque produces a change in the direction of the rotation axis is called precession. An example of precession is a spinning top, or a gyroscope. If the top is tilted so that it does not stand vertically, and it is not spinning, it will immediately fall to the ground when released due to the pull of gravity. However, when it is spinning, it does not immediately fall to the ground but instead precesses – it moves sideways. How can we explain this?

First we note that the force of gravity acting on the center of mass of the off-center top will provide a torque on the top. We also note that Newton's Second Law for rotational motion is a vector equation:

$$\Sigma\vec{\tau} = \tfrac{d\vec{L}}{dt}$$

Newton's Second Law for rotational motion tells us that the direction of the net torque will be the same direction as the **change** in the angular momentum vector.



The direction of the torque due to the gravitational force on the top in the picture will be out of the page (do the right hand rule to convince yourself of this). If the top is not spinning, its initial angular momentum is zero. An instant later, it will have an angular momentum pointing in the same direction as the torque due to gravity, which is out of the page. This means that the top's center of mass moved counter-clockwise (as seen from the picture), or down towards the floor.

If the top is spinning, its initial angular momentum is not zero; it is pointing along the direction of the axis of rotation (up and to the left in the picture). An instant later, the angular momentum will point in a direction such that the difference between the final angular momentum and the initial angular momentum is in the direction of the net torque. Another way of saying this is that the direction of the angular momentum will change in the direction of the net torque, which is out of the page in this case. The final angular momentum will point diagonally (not vertical nor horizontal) and slightly out of the page. If we repeat this analysis for the following instant in time, and the one after that, and the one after that, we see that the resulting motion of the top is for its center of mass to move in a horizontal circle (around the vertical axis). This is precession.

We can use a similar analysis to figure out in which way we need to push or pull a rotating object to make it do what we want it to do. If we want the direction of the axis of rotation to change in a particular way, we need to apply a torque in the direction of the desired change. This means that we need to apply a force in a direction and at a location such that the torque has the desired direction.

Since torque is perpendicular to both applied force and distance from the axis of rotation to the point of application of the force, the **force we apply must be perpendicular to the direction of the desired change!** This is counter-intuitive, but it really works out this way, and you will have a chance to explore this in lab this week.

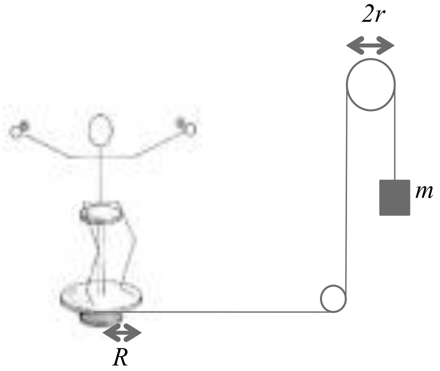
Setup for measuring moment of inertia

NOTE: This is at least a two-person job! Do not try to do this yourself, since you can get hurt.

DANGER: The hanging mass of 1kg is not too heavy, but you still don't want it falling on you or your belongings. Before releasing the 1-kg mass, make sure its path is clear and that it will not fall on top of anything or anyone.

The diagram of the setup is shown on the right. To set up the experiment, one person should hold the 1-kg mass while another winds the string to the bottom of the rotating platform.

NOTE: you and pulleys not drawn to scale



Things to look out for:

- Make sure the string goes smoothly through the groove of the bottom pulley
- Make sure the string does not have any knots, and is tense throughout
- Make sure the string goes smoothly through the **outer groove** of the disk on the rotary motion sensor, and that the hanging mass is to the right of the disk
- Make sure nothing will get in the way of the hanging 1-kg mass as it falls
- Make sure nothing will get in the way of the person rotating on the chair
- Check to make sure Logger Pro is playing nicely with the sensor (see FAQ below)

When you are ready to collect data, zero the sensor. If you set things correctly, the angular displacement of the disk should be negative.

Start collecting data, and release the hanging mass. Stop collecting once hanging mass reaches the floor.

Repeat as necessary until you have a good data run, with smooth data collection (string does not get stuck, string does not leave the groves on pulleys, person on rotating stool does not get stuck or knocked over, etc). Make sure you save the data run so you can do analysis later.

Rotating bicycle wheels

It is a bit difficult to describe the experiments you will be doing to explore conservation of angular momentum, so here are a few pictures.

For the first part, spin the bicycle wheel then sit on the stool. Hold the wheel as shown in the picture on the left, with one hand on either handle, and the wheel vertical. The wheel should be rotating such that the top is moving away form you (counter-clockwise in the picture).

You will be asked to turn the wheel over to that it is horizontal, or to sit on the stool holding the wheel horizontal. This is picture on the left. For this part of the lab, make sure you hold the wheel with both hands.

Measuring a parameter as a function of time that is supposed to be constant (with uncertainty)

Sometimes you can make many measurements very quickly. If during your analysis you calculate a parameter that you predict will be (and want to check if it actually is) constant, you will effectively have many separate measurements of this parameter. The best value for this parameter will be the mean of all these measurements, and the uncertainty will be the standard error. To get these, use the following procedure:

- Make a plot of the parameter as a function of time
- Select a region for which parameter looks reasonably constant. Since the parameter was calculated from your measured data, there will likely be a lot of noise, particularly at the beginning and end of data collection. This is to be expected, since any noise in your measurement will get amplified by the numerical calculations. This is why it important that your original data be as smooth as possible.
- Click on the statistics button (see logger pro help file). This will add a box on your graph that gives you the mean value of the selected points as well as the standard deviation and the number of points, which you can use to calculate the standard error.

FAQ for lab 4

Logger Pro doesn't see any sensors!

Logger Pro might not recognize the rotary motion sensor. If this happens, go to the menu bar and select "Experiment", then scroll down to "Set up sensors …" A new window should open. Select the channel where your sensor is connected, and scroll down to the name of the sensor (for example, "Rotary Motion"). Logger Pro should recognize the sensor after this.

Where is the "Collect" button?

If you had to manually tell Logger Pro that the sensor was connected, you might not see a "Collect" or a "Stop" button. To begin collecting data manually, go to the main menu bar on Logger Pro and select "Experiment" then scroll down to "Start Collection" (it's the first item on the list). Data collection begins immediately after you click on "Start Collection".

Data will be collected for the duration of the run, which is 10 seconds by default. If you wish to stop data collection before then, go to the menu bar, select "Experiment" and scroll down to "Stop Collection" (if data is being collected, it's the first item on the list). Data collection will stop immediately after you click on this item.