Background and Introduction

In this lab, you'll explore the oscillations of a mass-spring system, with and without damping. You'll see how changing various parameters like the spring constant, the mass, or the amplitude affects the oscillation of the system. You'll also see what the effects of damping are and explore the three regimes of oscillatory systems—under-damped, critically damped, and over-damped.

Harmonic motion

Most of what you need to know about harmonic motion has been covered in the lectures and in Kesten and Tauck Chapter 12, so we won't repeat it in depth here. The basic idea is that simple harmonic motion follows an equation for sinusoidal oscillations:

$$x_{undamped} = A\cos(\omega t + \phi)$$

For a mass-spring system, the angular frequency, ω, is given by

$$\omega = \sqrt{\frac{k}{m}}$$

where m is the mass and k is the spring constant. Note that ω does not depend on the amplitude of the harmonic motion.

Damping

The situation changes when we add *damping*. Damping is the presence of a drag force or friction force, which is non-conservative; it gradually removes mechanical energy from the system by doing negative work. As a result, the sinusoidal oscillation does not go on forever. Mathematically, the presence of the damping term in the differential equation for $x(t)$ changes the form of the solution so that it is no longer a simple sine wave.

Kesten and Tauck Section 12-7 describes the effects of damping. Essentially, the system will be in one of three regimes, depending on the amount of damping. Small damping causes only a slight change in the behavior of the system: the oscillation frequency decreases somewhat, and the amplitude gradually decays away over time according to an exponential function. This is called an *under-damped* system. The solution to the equation of under-damped systems is:

$$x_{underdamped} = Ae^{-\gamma t}\cos(\omega' t + \phi)$$

The parameter γ indicates how rapidly the oscillation decays and the parameter ω' is the new oscillation frequency:

$$\gamma = b / 2m$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\omega^2 - \gamma^2}$$

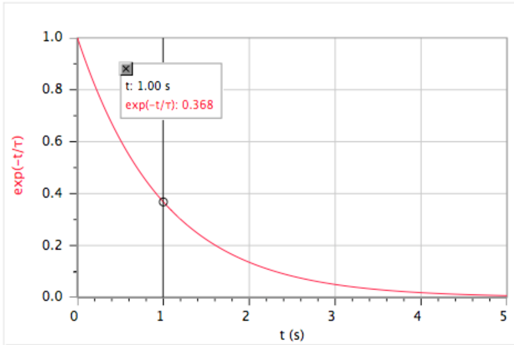
Here b is the damping constant. So for small damping (low b), the decay is very slow (small γ), and ω' is only slightly less than the un-damped angular frequency ω.

The "Underdamped" fit function in Logger Pro is a cosine wave whose amplitude decreases exponentially with time: position = A*exp(-t/B)*cos(C*t+D)+E. Note that the fit parameter B is the inverse of the parameter γ.

The key difference between this and un-damped motion is the exponential factor, exp(−t/B). The fit parameter B is called the *time constant* of the motion; it represents how quickly the amplitude decreases. The time constant is usually denoted with the Greek letter tau (τ), but Logger Pro doesn't do Greek letters in curve fitting, so that's why it's B in the equation.

Quantitatively speaking, τ is equal to the amount of time it takes for the amplitude to decrease by a factor of $1/e$, where e is the constant 2.718, the base of the natural logarithm. ($1/e$ is about 0.37.)

Here's a graph of exp(−t/τ) for τ=1.00 second:



If the damping is very large, the system does not oscillate at all. In fact, if it is displaced from equilibrium, it takes a long time for it to even return to its initial position because the drag force is so severe. Such a system is called *over-damped*. It turns out that this behavior is very nearly described by a simple exponential function:

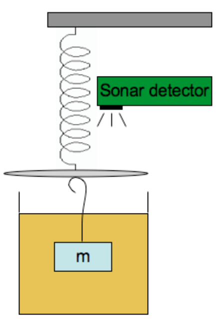
$$x_{overdamped} = Ae^{-\gamma t}$$

Here, the parameter γ depends on the damping coefficient b **in a different way** than for under-damped systems. For over-damped systems, γ is always less than ω, the angular frequency of un-damped oscillation.

The transition from over-damping to under-damping and vice versa is known as *critical damping*. As with over-damping, a critically damped system does not oscillate, but it returns to equilibrium faster than an over-damped system. It also follows (approximately) the negative exponential, but with a larger value of γ, which allows it to return to equilibrium faster than an over-damped system.

In fact, this is the defining characteristic of critically damped systems: **they return to equilibrium quickly and stay there**. An over-damped system is slow to return to equilibrium because it's just slow, period. An under-damped system gets to equilibrium quickly, but overshoots it and keeps oscillating about it, albeit with a gradually diminishing amplitude. This is the reason critical damping is interesting: in many applications (e.g. shock absorbers), you'd like any oscillations to damp out as quickly as possible.

<u>Exploring damping due to viscous drag</u>



To set up the experiment for damping with viscous drag, pour approximately 300 mL of corn syrup into the beaker and set it on the lab jack. Position the mass over the beaker and raise the height of the jack until the mass is resting in the center of the beaker as shown on the picture at left. You can also adjust the height of the bar holding the spring.

Make sure the mass is fully submerged. If it is too far down, the motion will be hindered by the bottom of the beaker; if it is too close to the surface, the surface tension (we'll talk about surface tension later in the semester) of the syrup will hinder the motion.

Wait until the system is fully at equilibrium.

Make sure you zero the detector before taking data.

Note that hot water dissolves in corn syrup a lot easier than cold or warm water.

Making a graph using data on page 2

If you want to make a graph but the data you want to plot do not appear on the list of variables, check to see if the variables that do appear are all on the same page and the ones that do not appear are on a different page. If this is the case, you can make the data from the different page appear by changing the variable in the x-axis.

- Double-click on the graph you want to change, or go to "Options" on the menu bar and scroll down to "Graph Options"
- A new window will appear.
- Select the tab "Axes Options"
- At the bottom left, select which variable you want on the x-axis from the drop-down list next to the label "Column"
- The variables that appear on the y-axis column will be the variables associated with the variable you selected for the x-axis column.
- When you are done, click "OK"