Background and Introduction

In this lab, you'll explore the physics of fluids, both through static properties (e.g., pressure and buoyancy) and phenomena related to fluid flow (e.g., viscosity and drag). You'll measure your own blood pressure and you'll also explore drag forces on a falling sphere in a viscous liquid and use the concept of terminal velocity to characterize the fluid's viscosity and density. Finally, you'll learn how to estimate Reynolds numbers and how they can be used for modeling purposes.

Static pressure and buoyancy

In a static column of fluid, the pressure in the fluid increases with increasing depth. For a fluid of density ρ in a column of height h, the pressure difference between the top and bottom of the column is:

$$P_{bottom} - P_{top} = \rho g h$$

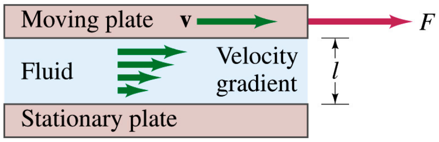
Due to this pressure difference, an object submerged in a fluid appears to weigh less than it would in a vacuum, because the fluid pressure pushing up on the bottom of the object exceeds the pressure pushing down on the top. This difference results in a net upward force called the buoyant force. Archimedes's principle states that *the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object.* There are two important cases to consider:

1. For an object that is *completely submerged* in the fluid, it displaces an amount of fluid equal to its own volume. Therefore, the buoyant force is equal to the density of the fluid multiplied by the volume of the object multiplied by g. This is true whether the object is static or moving in the fluid.

2. For an object that is *partially submerged* in the fluid, the volume of fluid displaced is only a fraction of the object's total volume. However, if the object is known to be in static equilibrium and there are no forces acting on it (other than buoyancy and gravity), then the buoyant force must be exactly equal and opposite to the weight of the object. Thus *a static floating object displaces an amount of fluid equivalent to its own weight.*

Viscosity

Viscosity is a property of a fluid that opposes relative motion. You can think of viscosity as being due to the frictional force between adjacent layers of fluid as they slide past each other. For a more technical definition, consider the following situation: two parallel plates of area A are separated by a fluid of thickness l:



You now want to move the top plate while keeping the bottom plate fixed. Due to viscosity, you need to apply a force F to the top plate in order to move it at a constant speed v. The layers of fluid between the plates will have different velocities: the layer at the very bottom doesn't move at all, and the layer at the top moves at the same speed v as the moving plate with a velocity gradient in between.

For many fluids, over a wide range of temperatures and pressures, the amount of force F that you need to apply is proportional to the speed v at which you want to move the plate and to the area A of the plates and inversely proportional to the distance l between the plates. The **viscosity** of the fluid is then *defined* to be the proportionality constant:

$$\eta = \frac{Fl}{vA}$$

The viscosity is represented by the Greek letter eta (η). Fluids that obey this simple proportionality are called *Newtonian* fluids.

Viscosity has dimensions of [M] / [L][T]. The SI unit of viscosity is the N·s/m^2 or Pa·s (Pascal-second). At room temperature, the viscosity of water is about 10^{-3} Pa·s. (However, this number can change by a factor of two with only a few degrees' difference in temperature.) The viscosity of air is about 2 x 10^{-5} Pa·s, although it, too, depends on the temperature.

Drag forces and terminal velocity

Objects that are moving in a fluid medium experience drag forces that oppose their motion, much like friction. Unlike our model of friction, however, the magnitude of the drag forces is *velocity-dependent:* the drag increases as the object's speed relative to the fluid increases. (Kinetic friction, by contrast, is generally taken to be a constant if the normal force is also constant.)

There are two kinds of drag forces, pressure drag and viscous drag. **Pressure drag** is due to the fact that the fluid has mass, and in order to move through the fluid, you have to push the fluid out of your path. The magnitude of the pressure drag on an object of cross-sectional area A moving at speed v through a fluid of density ρ is:

$$F_{\text{pressure drag}} = \tfrac{1}{2} c_D \rho A v^2$$

where c_D is a dimensionless constant called the *drag coefficient,* which can depend on the shape of the object and roughness of its surface. Typical values of c_D are about 0.1 to 1.

Viscous drag is due to the fact that fluids have viscosity, which opposes shearing of the fluid. For a solid sphere of radius r moving at speed v in a fluid of viscosity η, the viscous drag is given by the Stokes formula:

$$F_{\text{viscous drag (sphere)}} = 6\pi\eta r v$$

For shapes other than a sphere, the exact formula varies, but in all cases the viscous drag is directly proportional to speed, viscosity, and the linear size of the object.

For most situations, one kind of drag force is much greater than the other, so the smaller one makes a negligible contribution to the total drag on the object. For small objects, which are moving slowly, viscous drag dominates. For large objects, which are moving quickly, pressure drag dominates.

This raises the question of what size is considered to be "small" and what speed is considered "slow". The answer depends on the viscosity and density of the fluid. One way of looking at this is to consider the ratio of pressure drag to viscous drag. Since the area of an object is proportional to the square of its linear size ℓ, neglecting factors like ½ and π, we get (roughly):

$$\frac{F_{\text{pressure drag}}}{F_{\text{viscous drag}}} \approx \frac{\rho \ell^2 v^2}{\eta \ell v} = \frac{\rho \ell v}{\eta} = \text{Re}$$

The fraction $\rho\ell v/\eta$ is called **Reynolds number**, abbreviated Re. Because we got it by dividing one force by another force, the Reynolds number has no dimensions or units. When Re is much smaller than 1, viscous drag dominates; if Re is much greater than 1, pressure drag dominates. We'll see Reynolds number in lecture later in the semester. It turns out to be a very useful quantity for characterizing fluid flow.

One important thing to note is that the same fluid (i.e. the same ρ and η) can give you vastly different Reynolds numbers depending on the size and speed of the flow (ℓ and v). For example, an aircraft carrier moving through water has a Re of about 10^9; a swimming goldfish might have a Re of about 10^2; and a bacterium in the same water might have a Re of only 10^{-5}.

Conversely, if two flows have the same Re, then the physics in each is essentially the same regardless of the size, speed, or fluid involved. For example, a bacterium in water (with Re on the order of 10^{-5}) and a millimeter-sized bead in honey (Re also about 10^{-5}) behave very similarly. But you cannot model a bacterium in water by a macroscopic object moving in water at macroscopic speeds because the Re would be totally different. One situation is dominated by viscous drag, while the other is dominated by pressure drag.

Consider an object under the influence of a constant external force (e.g. gravity) that is also subject to drag, either pressure drag or viscous drag. If the object is initially at rest, the drag force is zero, so there will be no force to oppose gravity and the object will accelerate downwards. However, as it accelerates, the drag force increases to oppose the downward motion, and the faster it goes, the larger the drag force gets. Eventually, the object will be moving fast enough that the drag force is large enough to *exactly cancel the external force*. When this occurs, the net force on the object becomes zero, and it stops accelerating, which means its velocity remains constant. This final velocity is called the **terminal velocity**.

Terminal velocity is extremely useful, because we know that *when terminal velocity is reached, the sum of the forces is zero*. That means, in this case, that the drag force exactly balances the external force. This fact enables us to explore the drag force. If we can measure the terminal velocity for several different values of the external force, we can determine how the drag force depends on velocity. This technique is much easier than attempting to vary *v* and measure the drag force directly.

<u>Blood pressure sensor</u>

The blood pressure sensor consists of an inflatable cuff that connects to a pressure sensor, which is the small box that connects to the computer and interfaces with Logger Pro.

Blood pressure is typically measured with two numbers: the systolic and diastolic pressures. This is because your blood is not a static fluid: the pressure in it varies during each pulse beat as blood is pumped through the body. Both systolic and diastolic pressures are reported in units of mmHg (millimeters of mercury). A blood pressure of "120 over 80" means a systolic pressure of 120 mmHg and a diastolic pressure of 80 mmHg. The systolic pressure is the maximum pressure during a pulse, which occurs near the beginning of a cardiac cycle, while the diastolic pressure is the minimum pressure during a pulse, which occurs during the resting phase of a cycle.

The cuff of the blood pressure sensor is inflated by pumping on the bulb end of the tube. Next to the bulb is a valve; pressing this valve releases the air from the cuff. The valve also contains a screw that can be turned to fine-tune the rate at which air leaks from the cuff. Turning the screw clockwise increases the leak rate; turning it counterclockwise decreases the leak rate.

The blood pressure sensor works by inflating the cuff to a high enough pressure to actually cut off blood flow in your brachial artery (inside your arm). The pressure transducer then measures the pressure inside the cuff as a function of time as the cuff gradually deflates by leaking air.

You should be able to see from your upper graph (Cuff Pressure vs. Time) if you zoom in or from the lower graph (Oscillatory Amplitude vs. Time), that there are small "blips" in the cuff pressure every time your heart beats. This is due to your heart trying to pump blood through the blocked artery. Since blood can't get through, there is a temporary accumulation of blood in your artery. Your artery slightly expands in volume because the blood is incompressible. This expansion occurs at the expense of the volume of the cuff, leading to a small increase in air pressure inside the cuff. When the heartbeat subsides, the pressure returns to its previous level.

<u>Measuring blood pressure</u>

- Make sure that the patient removes all outer layers of clothing (coats, jackets, sweaters) and rolls up shirt sleeves as far as possible. If the sleeve is too tight to roll up, it is also fine to place the cuff over one thin layer of clothing.
- Wrap the inflatable cuff around the patient's upper arm so that the prickly Velcro surface (and the label "INDEX ➔") face **outward**. Also, turn the cuff so that the two rubber hoses are on the **inside** of the patient's arm by the bicep.
- The bottom of the cuff should be about 2 cm above the elbow joint.

- When the patient is ready, start collecting data in Logger Pro. The patient must keep arms and upper body completely still throughout the measurement.
- Rapidly inflate the cuff (using full pumps rather than small quick pumps) until the pressure reaches 160 and then wait.
- As you take data, air will slowly leak out. You will see the pressure slowly decrease in the upper graph; the patient will feel (and maybe even hear) the pulses in his/her arm. After about 40 seconds, the oscillatory "peaks" will appear in the lower graph. These peaks are used by the software to calculate systolic and diastolic pressures.
- The lower graph (Oscillatory Amplitude vs. Time) shows you the cuff pressure, except it subtracts off the overall decreasing trend of the cuff pressure as the air slowly leaks out of the valve.
- Eventually the lower graph will stop updating itself; at this point the data collection is complete. You can read off the systolic and diastolic pressures from the meters on the screen.

If there is no reading after 120 seconds, or a clearly meaningless result (e.g. systolic less than diastolic), you can try again. One common problem is that the cuff pressure should leak at a rate between 2 and 4 mmHg per second. If it is leaking too slow or too fast, you might not get a reading. You can adjust the leak rate using the screw on the release valve. Ask a TF for help doing this.

Measuring terminal velocity

Open the file "Lab6.cmbl". Logger Pro will prompt you on whether you want to set up sensors. Click the button for "Use file as is".

Set up the file to take video captures:

- Go to Insert on the main menu bar in Logger Pro.
- Scroll down and select "Video Capture…"
- Select the video camera on video input (not the computer's built-in camera)
- Select the default values for the resolution and the sound source.
- Click the "Options" button in the Video Capture window and set the following options:
    - Video Capture Only
    - Capture Duration: 20 seconds
    - Capture File Name Starts With: Teflon (or Titanium, or Steel, or Tungsten, depending on the material you are using for the specific run)
- When you are done, click OK.

Position the camera so that you can see the beaker of corn syrup. **Make sure you have something in frame in order to set the scale of your video.** In order to improve contrast, use a dark color object (like a book, or a piece of cardboard) on the background.

Using a pair of forceps, pick up a sphere out of the box and hold it in the corn syrup about an inch below the surface. Start collecting data, release the sphere and slowly remove the forceps from the syrup.

If during the video analysis you find that the bead's position does not change much between consecutive frames, you don't have to click through all the frames (that would take too long and be incredibly annoying). Click on the position of the bead, skip ahead a few frames, and click on the position of the bead in the new frame. Refer to the Logger Pro help file for more details on video analysis.

When you are done using a bead, fish it out of the corn syrup container with the forceps, rinse it in warm water (that's what the beaker is for) and put it away in the appropriate box.