Lab 2: Fourier Optics

This week in lab, we will continue our study of wave optics by looking at refraction and Fourier optics. As you may recall from class, the Fourier transform gives us a way to go back and forth between time domain and frequency domain. Here we will explore how Fourier transforms are useful in optics.

Learning Objectives:

In this lab, students will:

- Learn what a Fourier transform is and how it arises in optical systems:
  - Lenses
  - Diffraction gratings.
- Learn how an image can be modified through its Fourier transform.

Part A. Diffraction and refraction, simulated

In the last lab, we found that Huygens' principle was useful for understanding two-slit diffraction. Here we'll explore how Huygens' principle helps us understand *refraction* as well as diffraction from other arrangements of slits. Simulations can be a helpful tool for visualizing these phenomena. For the following exercises, use the wave simulator found at http://falstad.com/ripple.

1. Explore interference patterns from point sources.
    - Start with Setup: Single Source. Familiarize yourself with the settings. Change frequency, phase, damping, etc.
    - Try two sources one frequency. Observe interference from point sources. Move the points farther apart and see how it changes the interference pattern.
    - Try four sources, then six sources (at one frequency). Arrange six sources to approximate a plane wave. Arrange six sources to focus waves at a point.
    - Vary the phase of the emitters and note what happens.

2. Use the simulator to convince yourself that a plane wave can be approximated by a point source.

    - Change the Setup to Single Slit, Double Slit, and Triple Slit. Observe how a plane wave acts like a spherical wave as it passes through the slits.
    - Play around with the settings (frequency, damping, add borders or walls) and make observations of the interference patterns in the system.

3. Select "Setup: slow medium" or "Setup: refraction" and investigate what happens when a wave meets a medium with a different wave velocity.

4. List (or sketch) at least three things you learned from the simulation in your lab notebook.

You can see that we can create waves by arranging point sources in a certain way. This is also done in technological applications like antenna arrays where you want to create a radio wave going in a certain direction. In ultrasound probes of the body an array of acoustic sources is used to create a wave that focuses at a certain point.
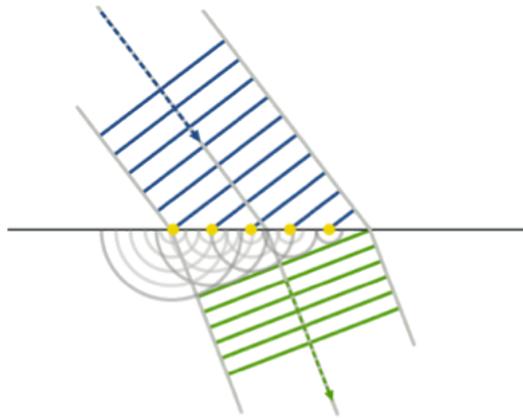
Figure 1: Wave refraction, according to Huygen's principle. (Image credit: http://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle)

Part B. Refraction and lenses

Making an image using refraction

Lenses rely on refraction to bend light and make an image. Use a converging lens to create an image of a filament on the screen or wall. Play around with lenses of different focal lengths. Vary the distance between the object and the lens, what happens to the image distance. What relationships do you notice between the image distance, the object distance, and the focal length?

Ray tracing

Ray tracing is a useful tool that can help you visualize where the image is formed in a lens system. There are three rules for tracing rays (see figure below):

1. A ray goes in parallel to the x-axis, and is refracted through the focal length of the lens.
2. A ray goes in through the focal length, and is refracted parallel to the x-axis.
3. A ray that goes through the center of the lens is not refracted.
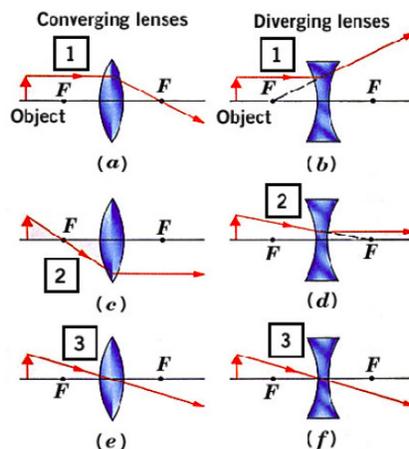
Image credit: http://www.physics.brocku.ca/courses/1P22_Crandles/lightoptics.php

The image is formed at the point where the rays intersect. In the case that the rays diverge, you can trace the ray backwards (as shown using the dotted lines above) to represent a "virtual" ray. If the point where the rays intersect includes a virtual ray, then you have created a virtual image. If you draw your lines to scale, you can use this method to accurately predict the location of the image, as given by the thin lens equation.

Part C: Diffraction with the Spatial Light Modulator

In this apparatus, we have a laser that illuminates a Spatial Light Modulator (SLM) used in video projectors. It is a transparent object like a slide that displays the images on the computer screen. The optics following the SLM focus the image onto a screen. In this case, the "screen" is a camera (with the optics removed) that projects the image onto the second computer monitor. The neutral density filter in front of the camera can be used to control the intensity of the image. The schematic below shows the basic elements of this system.
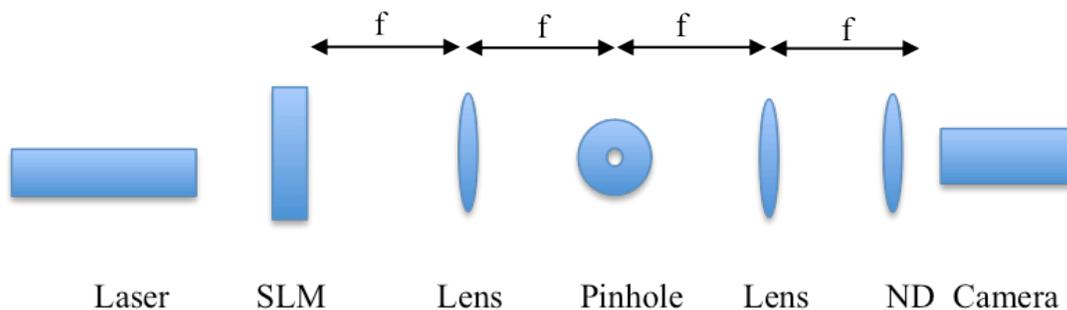

Figure 2: Fourier Optics System

First, make sure you understand the arrangement of the lenses and optical elements. What is imaged on the camera when the third lens (the one right in front of the camera) is not in the optical train? Can you draw a ray diagram to illustrate?

Pick one or two of the following three exercises. Be prepared to share your observations with the class. The number and width of slits can be controlled using the Matlab file on the computers.

1. Examine a single slit and vary the width. Observe how the distance between minima changes as width varies.
2. Examine a double slit and vary the spacing. Observe how the minima and maxima change as spacing varies.
3. Examine a double slit and vary the number of slits. What changes as the number of slits varies?

Part D: Fourier Optics

Now start by looking at a grating or a square wave (black and white bars). The grating is generated by Matlab code with variable number, width and spacing. Start with 15 slits, width of 20 pixels, spacing of 40 pixels. Vary the spacing and the width and observe the results.

We will think about these patterns in terms of Fourier transforms. We saw in class that the Fourier transform is useful for transforming a function of time to a function of frequency. One consequence of this is that periodic functions can be decomposed into a sum of sine waves. In general, the equations for a Fourier transform are given by:

$$f(t) = \frac{1}{2\pi} \int F(\omega)e^{-i\omega t}d\omega$$

$$F(\omega) = \int f(t)e^{i\omega t}dt$$

Try to understand the diffraction pattern that you see with a grating in terms of a *spatial* Fourier transform.

Once you understand the Fourier relationship (discuss with your TF), try some other experiments.

- Search for other pictures on the web and look at their Fourier transforms. Black and white, high contrast images work best. For instance, what is shown in <u>this image</u>?
- In general, what kinds of diffraction patterns do you see when you put an image of a crystal, or some other ordered pattern?
- Manipulate some images by blocking certain rays using the aperture and the pin. What are the effects? How can we understand these effects in terms of Fourier transforms?

References

E. Hecht and A. Zajac, *Optics* (Addison-Wesley, Reading, 1975) Sections 11.1 - 11.3.2 and 14.1 - 14.1.3.