Experiment 2: Conservation of Momentum

- **Learning Goals**

After you finish this lab, you will be able to:

1. Use Logger Pro to analyze video and calculate position, velocity, and acceleration.

2. Use the equations for 2-dimensional kinematics to calculate the speed of a projectile.

3. Determine how uncertainties in individual measurements "propagate" into uncertainties for calculated quantities.

4. Use your video measurements, kinematic analysis, and skills in error propagation to determine whether momentum is conserved for the "collision" involving the Gauss Gun.

Introduction: Please read all of this BEFORE you come to lab.

- **Basic error propagation**

Chapter 2 of Taylor gives a few formulas for propagation of error, which is the process by which we can determine the uncertainty of a quantity which was not itself measured directly, but instead calculated from one or more measured values. Error propagation answers the question, "how do the uncertainties in the measured values propagate to the uncertainty in the final answer?"

The simplest way to answer this question is also the crudest, but it has the virtue of being quite straightforward to understand and apply: assume the worst-case scenario in each direction, and look at the range of values you get for a final answer.

For example, suppose I have measured the values $A = 22 \pm 3$ and $B = 15 \pm 2$, and I am interested in the quantity $C = A - B$. The best guess value of C is just the mean value of A minus the mean value of B, which is $22 - 15 = 7$. Now I can estimate the uncertainty by looking at the possible values of A and B that will make C as big as possible, and as small as possible. To make C as big as possible, A needs to be as big as possible (25) and B needs to be as small as possible (13), so C could be as big as $25 - 13 = 12$. Similarly, C could be as small as $19 - 17 = 2$. The range of values of C is from 2 to 12, so the "worst-case" uncertainty for C is 5, and I would say that $C = 7 \pm 5$.

No matter how complicated the expression is for the final quantity whose uncertainty we want to know, this simple process will do the trick, although sometimes you will have to think carefully about whether you "want" A to be big or small in a given expression in order to make C big or small. The equations for the uncertainty of a difference and uncertainty of a product given in Taylor were derived from this method too. If you are comfortable with those equations, you can use them instead of applying the method directly. The two are entirely equivalent, but applying the formulas is probably faster if you understand how.

As you might imagine, the worst-case method tends to overestimate the actual uncertainty of the final quantity. (If A and B have independent random errors, then it would be pretty unlucky to have A be as large as possible at the same time that B is as small as possible.) But for now, the worst-case method is preferable because it is easy to understand and use, and it does give results that are quite close to the best answer.

- **Momentum**

You've already learned about momentum in the first few weeks of lecture, section, and homework. But here is a summary of the big ideas:

- The total momentum of an isolated system is conserved (it doesn't change).

- The momentum of a single object is its mass times its velocity. Momentum is a vector.

- The total momentum of a system is the vector sum of the momenta of each object in the system.

- An isolated system is one that has no interactions with anything outside of the system.

- A system can be considered to be functionally isolated if there is no net external force on the system.

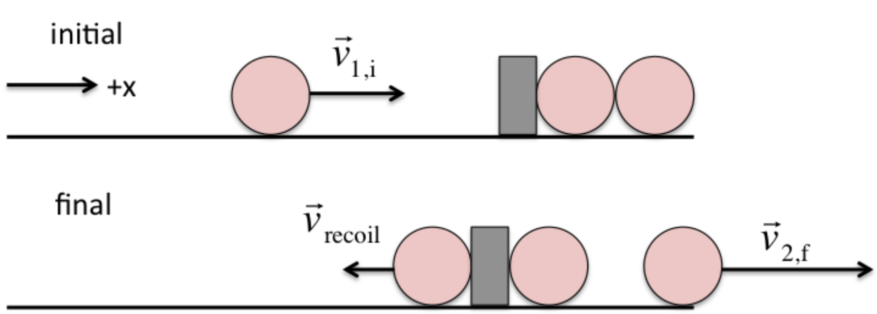
- The *impulse* on a system is defined as the change in the system's total momentum between some initial state and some final state (final momentum minus initial momentum).

    - For an isolated system, therefore, the impulse is zero no matter what the initial and final states are chosen to be.

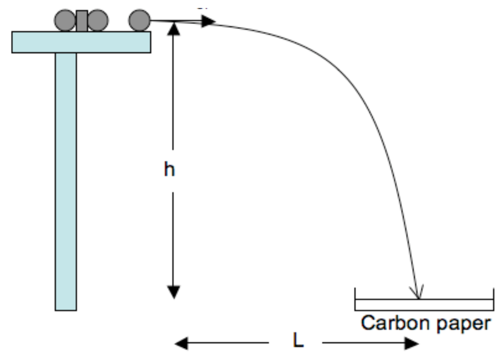
    - For a non-isolated system, the impulse on the system during some interaction is given by the average external force on the system multiplied by the time duration of the interaction.

- **The Gauss Gun: Is Momentum Conserved?**

The main goal of this lab is to determine whether the momentum of the balls and magnet are conserved during the firing of the Gauss Gun. In other words, can we treat the Gauss Gun as a *functionally isolated system* during the interaction? You already saw the Gauss Gun in lab 1:



You have 3 identical ball bearings (mass m_{b}) and a magnet (mass m_{m}). Initially, a ball is moving with velocity $\vec{v}_{1,\mathrm{i}} = v_{1,\mathrm{i}}\hat{x}$ towards the stationary magnet-ball-ball complex. Just after they collide, the ball-magnet-ball complex recoils with velocity $\vec{v}_{\mathrm{recoil}} = -v_{\mathrm{recoil}}\hat{x}$, while the last ball flies off with velocity $\vec{v}_{2,\mathrm{f}} = v_{2,\mathrm{f}}\hat{x}$.



You will measure the speed of the incoming ball v_{1i} and the speed of the recoil v_{recoil} using a video camera and the video analysis tools in Logger Pro. But the outgoing ball will be moving too quickly to catch it on the video.

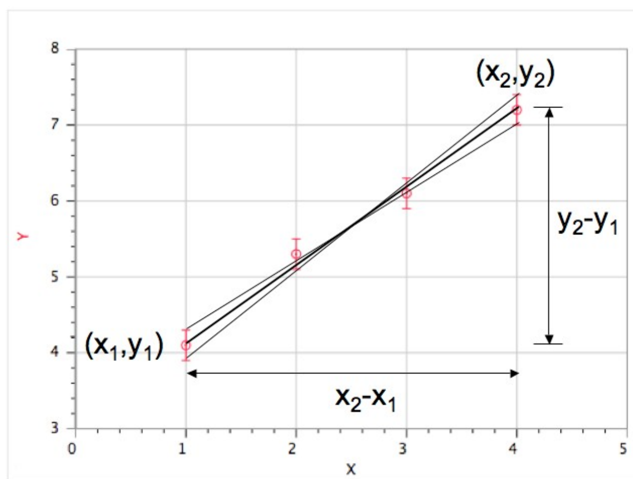
So you will measure the speed $v_{2,f}$ of the ball that flies off by seeing where it lands on the floor (using carbon paper, just like you did in Lab 1). You will need to find an equation for the speed $v_{2,f}$ of the launched ball in terms of the length L that it travels and the height h from which it was launched.

Your ultimate goal will be to calculate the *initial* momentum of the entire Gauss Gun (magnet plus all three balls), find the *final* momentum, and compare the two. You will need to **propagate the uncertainty** of the measurements of mass, speed, and length to estimate the uncertainties of the momentum. As we discussed in Lab 1, if you want to know if two quantities are equal, you need to know the *uncertainties* of these quantities.

- **Calculating the uncertainty of a slope**

As with individual measurements, we can *approximate* the uncertainty associated with linear fit parameters. A simple way of doing this is to draw the steepest and shallowest lines that are consistent with the error bars on the points, and calculate the slope for each.

If there are vertical error bars the three lines (best fit, steepest, and shallowest) might look like the graph shown at right:



We can approximate the uncertainty in the slope as:

$$\delta m \approx \frac{m_{max} - m_{min}}{2}$$

Here δm is the uncertainty in the slope, m_{max} is the slope of the steepest line (found by using the lowest possible value for the first point and the highest possible value for the last point), and m_{min} is the slope of the shallowest line (found by using the highest possible value for the first point and the lowest possible value for the last point).

Experiment 2: Lab Activity

- Follow along in this lab activity. Wherever you see a question highlighted in red, be sure to answer that question, or paste in some data, or a graph, or whatever is being asked. Your lab report will be incomplete if any of these questions remains unanswered.

Who are you? Take a picture of your lab group with Photo Both and paste it below along with your names.

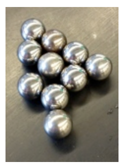
- **Tutorial:**

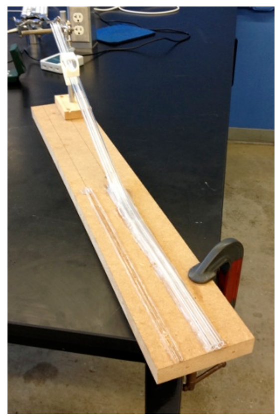
  Before setting up the equipment, work through the Logger Pro tutorial number **12: Video Analysis** (in the Tutorials folder under Experiments).

- **Materials:**

  Many of the materials are the same as in Lab 1:

  A magnet: 

  Some steel balls: 

  A long track with a ramp: 

  You'll also have some blank sheets of paper, some carbon paper, and a 2-meter stick.

  In addition, for this lab you'll also have a **digital video camera** on a tripod, a **level** so you can make sure that your Gauss Gun is level, some **lubricant** to reduce friction on the track, and a **balance** so you can measure the masses of the balls and the magnet.

- **Set up the experiment**

  Set up the Gauss Gun as you did in Experiment 1: place it right at the edge of the track. Release a ball near the bottom of the ramp so it strikes the magnet **slowly** and causes the Gauss Gun to fire. Take note of where the launched ball lands on the floor, and tape a sheet of white paper in that spot. Then place a sheet of carbon paper (ink side down) on the paper where the ball will land. Set up the video camera so you can capture the Gauss Gun clearly. Be sure that the camera can capture the **entire** recoil of the magnet system until it comes to rest.

  Take a few minutes to play around with the setup and plan how you will arrange the various pieces (video camera, balls, magnet, track, carbon paper). You'll only have time to analyze a single run, so be sure everything is set up correctly. A very small amount of lubricant (just a drop or two) on the track will help reduce friction.

If Logger Pro is still open from the tutorial, quit it now. Then make sure that the camera is plugged in, turned on, and connected to the computer, and open Logger Pro.

Go to the "Insert" menu and select "Video Capture." If the settings are not visible, click "Show Settings." For the "Video Input" select **ZR830**. The video resolution should be set to 720 x 480. Set the "Video Compression" to H.264 SD. Then click on "Options" and set the Capture Duration to 20 seconds.

When you are ready to record, tape a fresh sheet of white paper to the floor, put the carbon paper in place, and be sure that the balls are all set and aligned. Then click Start Capture. Release ball 1 near the bottom of the ramp so that it rolls **slowly** towards the magnet, and watch the collision closely.

When the video capture finishes, make sure all of the following things are true:

- Ball 1 approaches the magnet smoothly (no bouncing or wobbling as it rolls).

- Collision does not impart any significant y-velocity (vertical motion) to any part of the system (no bouncing).

- Recoiling magnet system comes to rest on the track, and the entire recoil (until everything comes to rest) is recorded in the video capture.

- You can identify which carbon paper mark was left by the ball during your actual run.

If any of the above is not true, delete the video capture and repeat the experiment. It doesn't take long to do, but it does take a lot of analysis, and you don't want to waste your time analyzing data from an incomplete run.

Remember, you don't need to be able to see ball 3 shoot off in the video capture. We expect that you won't, actually, because it goes too fast. That's why we have the carbon paper setup.

- **Analyzing the data**

Is momentum conserved? You will need to measure the momentum of the system before the collision and after the collision. The table below summarizes your results; you will fill in the relevant quantities as you go along.

Momentum before collision			Momentum after collision		
	Value	Uncertainty		Value	Uncertainty
Mass of ball 1			Mass of 1---M---2		
Mass of M---2---3			Mass of ball 3		
Vel. of ball 1			Vel. of 1---M---2		
Vel. of M---2---3			Vel. of ball 3		
Momentum			Momentum		

Before the collision:

- **Measure the relevant masses (with uncertainty!) and enter them in the table above.**

  The balance reads to the nearest hundredth of a gram. That means that when you measure a mass using the balance, the reading error on the mass is 0.01 g.

  If you use the balance to measure the mass of a magnet, you might get an inaccurate reading because of an attractive force between the magnet and the metal inside the balance. To get an accurate measurement, put an inverted plastic beaker on top of the balance and re-zero it before putting the magnet on top. That way the magnet will be several inches away from anything metal while it is being weighed.

- **Use video analysis to find the position of ball 1 before the collision as a function of time.** Remember, you'll have to set the scale for the video. **Use the length of the magnet-ball complex itself to set the scale**. Refer to the Tutorial named "12 Video Analysis" if you need a reminder on how to do this. You'll need to measure the length of the magnet-ball complex with a ruler:

Length of magnet-ball complex:

  - You might need to tilt your axes to make sure all the motion is in the x-direction. This can happen if your video camera is not aligned perfectly, and track is not quite horizontal. Click the "set origin" button in the video analysis window (the third one down from the top right corner). Click near the lower left corner of the video window and a set of yellow axes will appear. You can rotate the axes by dragging the big yellow dot up or down slightly. Adjust the axes until the x-axis is aligned with the Gauss Gun track.

  - Once you have done this, we are no longer interested in the y-motion (there shouldn't even be any y-motion), so change the settings on the graph so that it only shows you x vs t. (We also don't want it to show v_x or v_y. You'll make your own determination of velocity.)

- **Make sure you include error bars in your *x* vs *t* graph.** You may assume that the uncertainty in the x measurements is ±0.2 cm (2 mm).

  Use a linear fit to an appropriate section of the x vs t graph to determine the velocity of ball 1 before the collision. **Paste your graph of *x* vs *t*, with the fit, below.**

Insert graph here:

Velocity of ball 1 before collision:

Using the method described in the introductory materials, calculate the uncertainty in the slope that results from error bars of this size. This is the uncertainty in the velocity of ball 1. Be sure to write out your calculations clearly, take a picture using Photo Booth, and paste in your calculations below.

Uncertainty in velocity of ball 1:

Photo of the calculations you used to find the uncertainty in the velocity of ball 1:

Record the masses and velocities in the main data table above.

After the collision:

- **Measure the relevant masses** and record them (with uncertainty) in the main data table above.

  In the same video capture, you can track the motion of the recoiling 1-M-2 trio after the collision. However, unlike the incoming ball 1, we do not expect that 1-M-2 moves at constant velocity after the collision. It has some velocity right after the collision, but later it is observed to come to rest. So there must be a net force acting on it.

  Using the video analysis tools, add a new trail and plot the position of the 1-M-2 trio from the first frame after the collision (which we'll call t_0) until it comes to rest.

  The best model of the forces acting on the recoiling trio predicts that it will undergo *constant acceleration* from time t_c to t_r. **If the motion really is constant acceleration, what kind of curve will describe the x vs t graph?**

A:

Try fitting an equation of the form below and find the best-fit values of A, B, and C. (Note that t_0 is a constant—the time of the first frame after the collision—not a parameter to be fit.)

$$x = A(t-t_0)^2 + B(t-t_0) + C$$

Paste the graph, with fit, here:

What is the physical meaning of the constants A, B, and C in terms of the usual kinematic quantities? From the fit in the graph above, what was the initial velocity v_{recoil} of the recoiling magnet complex just after the collision (at time t_0)? Be sure to include units, and think carefully about the *sign* of the x-component:

$v_{\text{recoil,x}}$ =

Based on previous experience, we can estimate the **uncertainty** of this velocity v_{recoil} to be approximately 5 cm/s.

- Using the carbon mark left by ball 3 when it hit the floor, measure the quantities h and L from the figure in the introduction. Include an estimate of your reading errors.

h =

Uncertainty of h =

L =

Uncertainty of L =

Plug in your numbers above, and the accepted value of g, to the expression you found in the introduction, and get the x-velocity of ball 3 just after the collision:

Velocity of ball 3 =

Use error propagation to calculate the uncertainty in the velocity of ball 3 based on your estimated uncertainties in h and L. You may neglect any uncertainty in g. Be sure to write out your calculations clearly, take a picture using Photo Booth, and paste in your calculations below.

Uncertainty of velocity of ball 3 =

Photo of the calculations you used to find the uncertainty in the velocity of ball 3:

Calculate the x-component of the total initial momentum, **with uncertainty**, as well as the x-component of the total final momentum, **with uncertainty**. Write these here and in the table. Be sure to write out your calculations clearly, take a picture using Photo Booth, and paste in your calculations below.

$\boldsymbol{p_{ix}}$ =

Uncertainty =

$\boldsymbol{p_{fx}}$ =

Uncertainty =

Photo of the calculations you used to find the uncertainty in calculations of momentum:

- **Conclusion**

The big question! What can you conclude from your data about momentum conservation in the Gauss gun system?

What is the most important thing you learned in lab today?

What aspect of the lab was the most confusing to you today?