Background and Introduction

In this lab, you will be exploring the behavior of electrical components connected in circuits. The most basic thing to keep in mind is that nothing interesting will happen at all unless there is a *circuit*—that is, a closed loop where charge can flow.

<u>Ohm's law</u>

In many circumstances, the current i through a resistor and voltage ΔV across a resistor are proportional to each other. This empirical fact is known as Ohm's law.

The proportionality constant between ΔV and i is called the resistance, R. Ohm's law can be stated quantitatively as: $\Delta V = iR$.

Resistance is measured in ohms (Ω). An ohm is a volt per ampere. Because amps are so large, useful resistances are often on the order of kiloohms (kΩ) or megaohms (MΩ).

All real circuit elements have some resistance. However, when the circuit contains resistors or light bulbs, the comparatively small resistance of other circuit elements such as wires can often be neglected.

<u>Power</u>

The electrical power (rate of change of electrical energy per unit time) of a device is equal to the current through it multiplied by the voltage across it: $P = i\Delta V$. Electrical power is measured in watts (1 W = 1 Joule/second), just like mechanical power.

Being very careful with the sign of electrical power leads to some useful insights:

- When the potential *decreases* in the direction that current is flowing, ΔV is negative, so that P is also negative. That tells us that electrical energy is being *lost* in that circuit element. This is always the case with resistors.
- When the potential *increases* in the direction that current is flowing, ΔV is positive, so P is also positive. This is usually the case with a battery.

For resistive elements (resistors, light bulbs, heaters, etc.), the power is the rate at which electrical energy is being converted into heat or light. We often talk about power "dissipated" in a resistor or bulb; as a result, the minus sign is often dropped with the implicit understanding that electrical energy is being lost or dissipated.

For a battery, P is the rate at which the battery is *supplying* electrical energy, which the rest of the circuit can use to do work. It is also the rate at which the chemical energy stored in the battery is being depleted.

<u>Resistor</u>

Resistors are just circuit elements that have resistance. Resistors obey Ohm's Law, so the voltage across a resistor is always equal to the instantaneous current through it multiplied by its resistance. Recall that if no current flows through a resistor, there can be no potential drop across it.

- Resistors connected in series add: $R_{eq} = R_1 + R_2 + ...$
- Resistors connected in parallel add inversely: $1/R_{eq} = 1/R_1 + 1/R_2 + ...$

The power delivered to a resistor can be calculated by $P = i\Delta V = i^2R = \Delta V^2/R$ (all equivalent due to Ohm's law). This is the rate at which electrical energy is being converted to heat energy.

<u>RC circuits</u>
Circuits containing a resistor and capacitor are called RC circuits. RC circuits are time-dependent—that is, they are not static circuits. They asymptotically approach a steady-state limit in which all voltages remain constant and all currents go to zero. (Remember, a non-zero capacitor current means that the charge—and therefore the voltage—on the capacitor is *changing*.)

Any voltage in an RC circuit relaxes towards its final steady-state value exponentially with time; that is, the difference between $\Delta V(t)$ and its final value decreases as $\exp(-t/\tau)$, where τ (the Greek letter *tau*) is called the *time constant*. In a circuit with capacitance C and resistance R, the numerical value of τ is equal to R times C. If R is in ohms and C in farads, then the product RC has units of seconds. (1 Ω = 1 V/A; 1 F = 1 C/V; so 1 Ω·F = 1 C/A = 1 s.)

The meaning of τ is that it is the time required for the exponential factor to become 0.37, or 37% of its original value. (Plugging in $t=\tau$ gives exp(-1), which is just 1/i, or about 0.37.) For a voltage which is decaying to zero, τ is the time it takes for it to reach 37% of its initial value. For a voltage which is going from zero to some non-zero final value, τ is the time it takes to reach 63% of that final value (since the *difference* between ΔV and the final value decays to 37%).

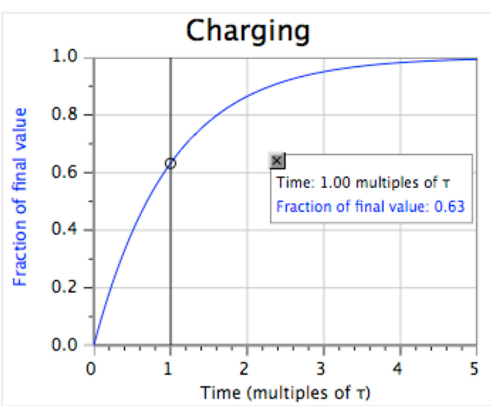
You can usually determine all of the "initial" (right after the circuit is completed) voltages by remembering that the total change in voltage ΔV if you go *all the way around the circuit in a complete loop* must be zero. So, for instance, the voltage will increase when you go through the battery, and decrease (according to Ohm's law) when you go through a resistor. Then keep in mind that *the charge on a capacitor cannot change instantaneously*. Thus the voltage across a capacitor also cannot change instantaneously; right after any switch is thrown, the capacitor must have the same voltage it had right before.

The fact that all currents go to zero in the final steady state makes it easy to determine the final values of all of the voltages. For a resistor, according to Ohm's law, if the current i is zero, then the voltage drop ΔV must also be zero.

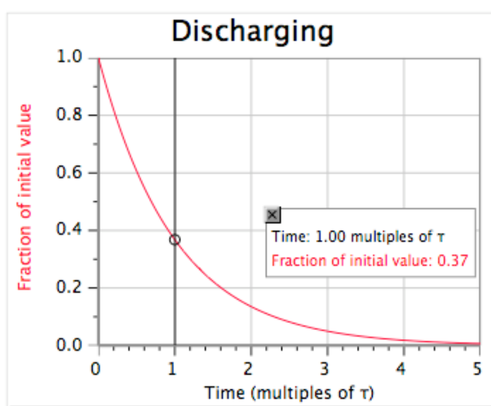
Once you know the initial and final values for some quantity X in an RC circuit (X can be a voltage, current, charge, whatever), then you can apply the exponential relaxation:

$$X(t) = X_f + (X_i - X_f)\exp(-t/\tau)$$

This equation expresses the idea that $X(t)$ starts at X_i at $t=0$ and ends up at X_f for large t. Also, either the initial or final value is almost always zero (*e.g.*, when charging a capacitor, it starts at zero; when discharging, it ends at zero), which simplifies the equation quite a bit:



$X_{charging}(t) = X_f\,(1-\exp(-t/\tau))$



$X_{discharging}(t) = X_i\,\exp(-t/\tau)$

Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

Determining the resistance of a resistor

The resistance of a resistor is indicated by the set of four colored stripes on the resistor. The first two bands taken together indicate the first two digits of the resistance, the third band is the multiplier (power of ten), and the fourth band is the tolerance.

For example, the following resistor has a resistance of 26 (red-blue) times 10^5 (green), within a tolerance of 5% (gold). So its resistance is 2.6 MΩ, give or take 5%.

If you are not sure which end to start from, most resistors have a gold or silver band as their fourth band to indicate tolerance.

Charging and discharging a capacitor

Open the Logger Pro file Lab2RCCircuit.cmbl. The file has been set up for you so that you should be ready to collect data. When you click on the “Collect” button, it will collect data for 10 seconds and then automatically stop. **Do not press the stop button while it is collecting**.

Familiarize yourselves with the RC circuit board. Make sure you know which switch position does what (and which component is the resistor and which is the capacitor!). It may be helpful to trace the circuit with your finger. Note that if you charge the capacitor *it remains charged*, even after you disconnect the battery, until you allow it to discharge.

You might notice a weird behavior when collecting data of the charged capacitor: when the capacitor is charged and the switch is in the neutral (neither charging nor discharging) position, the capacitor voltage seems to decrease over time very slowly. This is because of the voltmeter itself acts like a 10 MΩ resistor connected across the capacitor. Even when the rest of the circuit is disconnected, the capacitor can discharge through this large resistor.

To minimize any error caused by this slow discharge, keep the switch in the charging position until you are ready to throw it into the discharging position. It's okay if the voltage goes down a little bit before the discharging begins; the only difference is that it is as if the initial charge on the capacitor were somewhat lower.

With the capacitor fully charged, press collect and then when it says "Waiting for data," throw the switch to the discharging position.

I. Before you come to lab

- Read this write up and the background material.

II. Learning Objectives

In this lab, you'll gain familiarity with circuits by solving two "puzzles" involving measuring some unknown quantities. The key to solving these puzzles will be a good understanding of circuits, both on a theoretical and a practical basis. You will also investigate the behavior of an RC circuit as a function of time.

III. Materials

Differential voltage probe

The differential voltage probe is basically just a voltmeter that interfaces with Logger Pro. Like the multimeter you used in Lab1, it measures the potential at the red probe minus the potential at the black probe.

Selection of alligator clip leads

Battery pack

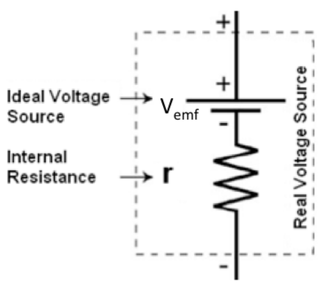
This is a pack containing two 1.5-volt batteries connected in series. It can be thought of as a single 3-volt battery. The red lead connects to the positive terminal of the battery; the black lead connects to its negative terminal.

Selection of resistors

The resistance of a resistor is indicated by the set of four colored stripes on the resistor. For an overview on how to read the value of the resistance from these colored stripes, refer to the supplemental.

1 "mystery" resistor

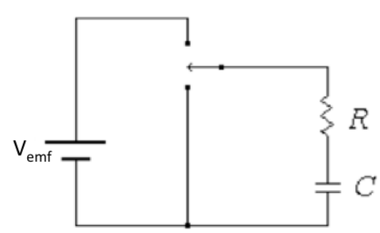
1 two-terminal "ER" black box



This is a box containing a battery (emf) and a small resistor, connected in series. The two exposed metal contacts A and B are the + and - terminals, respectively, of the diagram shown at left.

RC circuit board

This is a board wired with the circuit at right, where: R = 30 kΩ, C = 1 μF, and the battery is a standard 1.5-volt AA cell battery.



When the switch is in the up position, the battery is included in the circuit to charge the capacitor. When the switch is in the down position, the capacitor discharges through the resistor. When the switch is in the middle position, no current flows and any charge which happens to already be on the capacitor remains there.

V. Procedure

Who are you? (Picture, names, and emails please)

A:

Part 1: Mystery resistor

In this part of lab, you will attempt to determine the value of an unknown resistance by designing one or more circuits involving the mystery resistor and making appropriate measurements. Be forewarned: it's not trivial.

The tools you will be able to use to measure the mystery resistor are:

A battery pack
A selection of resistors of different (known) values
LabPro interface with differential voltage probe and the Logger Pro software. (Note that the only measurements you can make are *voltage* measurements. You cannot make direct measurements of current or resistance!)

Play around with the components at your disposal and see what you can measure. At some point, however, work with your lab partners to devise a systematic plan to determine the mystery resistance. Then put your plan into action!

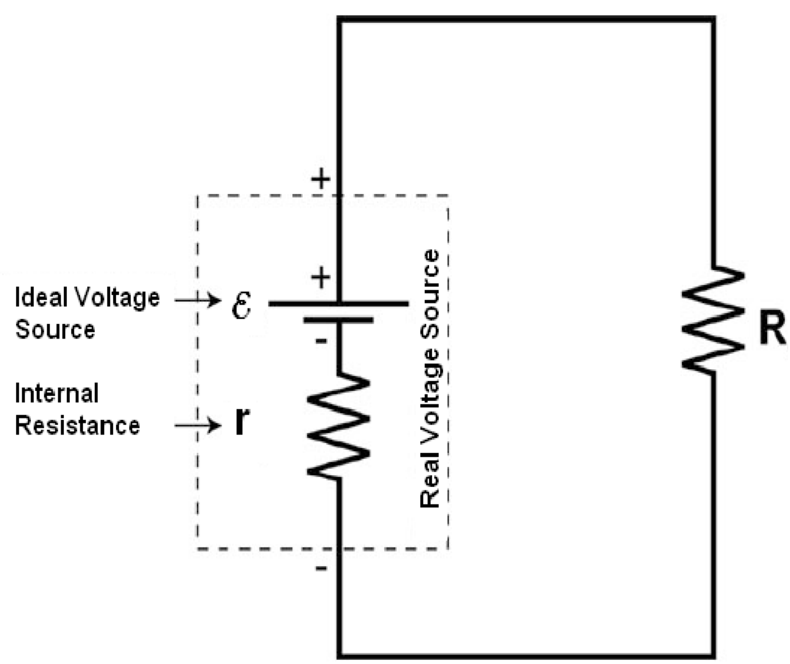
Describe your procedure here; be specific! Include snapshots of circuit diagrams if you need to. Feel free to add as many steps as you need.
A:

Record your measurements and calculations here. Include uncertainty measurements, since we will want to know the uncertainty of your final answer.
A:

What is the mystery resistance R? What is the uncertainty in your measurement of R?
R =
uncertainty in R =

Part 2: What's in this box?
In this part of the lab, you will determine the value of the the emf, V_{emf}, and the internal resistance, r, of a battery inside a black box . The black box, labeled "ER," contains a "battery" which consists of a voltage source and a small-but-not-tiny "internal resistance." The battery is oriented so that terminal A is at a higher potential than terminal B. As with the previous section, you will only be able to measure voltage, not current nor resistance!

To get you started, let's work through some calculations. A schematic circuit diagram of a real voltage source (the "black box") is shown below. The real voltage source can be modeled as an ideal voltage source along with an internal resistance *r* in series; the two cannot be separated (i.e. you can't poke around inside the dashed line with a voltmeter). However, you can calculate the internal resistance along with the EMF of the ideal voltage source by connecting an external resistor *R*, as shown.



a) You want to measure the voltage across *R*. Add the voltmeter to the diagram (at right) in the appropriate place with the correct connections. You can represent the voltmeter by a V with a circle around it.

b) Show that the voltage ΔV across the external resistor *R*, in terms of $\mathcal{E}$, *R*, and *r*, is given by the expression:

$$\Delta V_R = iR = \boxed{\left(\frac{\varepsilon}{R+r}\right)R}$$

(Hint: use the fact that the total change in voltage around a closed loop is zero.)

Now let's imagine changing the resistance of the external resistor *R*. Just to get a feel for how this works, fill in the following table with the values of ΔV for different values of *R* (in terms of *r*):

R	ΔV_R	$1/R$	$1/\Delta V_R$
r			
$2r$			
$3r$			
$4r$			

Plot ΔV_R vs R and $1/\Delta V_R$ vs 1/R separately for the data you calculated above. Do either of these plots look linear?

What are the slope and the y-intercept of the $1/\Delta V_R$ vs 1/R plot?

Now use what you learned from this little "model" exercise to devise a procedure to measure the unknown resistance *r*, using a series of known resistors *R*.

Describe your procedure and record your measurements here. Include uncertainty measurements, since we will want to know the uncertainty of your final answer. Feel free to add as many steps as you need:
A:

Record your results along with a copy of any graphs you used here. Don't forget uncertainties!
V_{emf} =
r =

Part 3: Charging and discharging of a capacitor
In this part of the lab you will use an RC circuit board to explore the processes of charging and discharging a capacitor. You will also find the time constant for this circuit. Make sure you familiarize yourself with the RC circuit and the LoggerPro file as described in the "Materials" and "Procedures" parts of the write up before taking data.

Charging RC
Start with the capacitor fully discharged. Connect the differential voltage probe in order to measure the voltage across the capacitor and click on Collect. When you see the message "Waiting for data," throw the switch into the charging position.

What are the "initial" and "final" values of the capacitor voltage?
A:

What functional form does the plot look like?
A:

How long after you throw the switch does it take for the capacitor voltage to go 63% of the way from its initial to final values?
A:

Is this consistent with the stated component values for R and C? Explain.
A:

Paste the relevant part of your graph here. Be sure to display the results of your fit:
A:

Discharging RC
Start with the capacitor fully charged. To minimize any error caused by slow discharge (see supplemental and Warm-up for explanation), keep the switch in the charging position until you are ready to throw it into the discharging position. It's okay if the voltage goes down a little bit before the discharging begins; it is as if the initial charge on the capacitor were somewhat lower. Click on Collect and when you see the message "Waiting for data", throw the switch into the discharging position.

How long does it take for the capacitor voltage to become 1/e of its initial value?
A:

Is the time constant the same as it was for charging? Explain.
A:

Paste the relevant part of your graph here. Be sure to display the results of your fit:
A:

VI. Conclusion
What is the most important thing you learned in lab today?
A: