Lab 2: Mach-Zender Interferometer Overview

Goals:

1. Study factors that govern the interference between two light waves with identical amplitudes and frequencies
    1. Relative phase
    2. Relative polarization
2. Learn to setup and align optical systems
3. Learn about Quantum Mechanical measurements

Process:

1. Setup a Mach Zender interferometer to observe wave interference at port1 and port 2
2. Vary the phase between the two arms of the interferometer and measure the associated change in the fringe patterns at port 1 and port 2
    1. Using voltage on a piezo electric crystal to alter path length
    2. Changing index of refraction inside a cell by pumping out the air
    3. Changing the division of the optical path between air and glass by rotating a microscope slide
3. Control the relative polarization of the two light beams and consider the effect on interference patterns at port 1 and port 2

Deliverables

1. Show us the fringes in a Mach-Zender interferometer, and demonstrate that you can make the fringes either horizontal or vertical. Optimize the alignment so that the spacing of the fringes is as large as possible
2. Measure the distance change as a function of voltage for the piezo electric crystal
3. Measure the index of refraction of air
4. Measure the index of refraction of a glass slide
5. Measure interference patterns as a function of the relative polarization of the two interfering beams
6. Measure the index of refraction shift due to heat, butane, and LCD
7. Measure and optimize fringe contrast

Slide Outline

1. **1. Introduction to Mach –Zender Interferometer (slide 3)**
2. **Optics Discussion**
    1. **General review of interference between two waves with the same amplitude and frequency (slides 4-5)**
    2. **Review of intensity as a measured quantity (slide 6)**
    3. **Review of plane wave interference (slides 7-8)**
    4. **Explanation of fringes observed in Mach-Zender interferometers (slides 8-10)**
3. **Deliverables**
    1. Show us the fringes in a Mach-Zender interferometer, and demonstrate that you can make the fringes either horizontal or vertical. Optimize the alignment so that the spacing of the fringes is as large as possible (slide 11)
    2. Measure the distance change as a function of voltage for the piezo electric crystal (slide 12)
    3. Measure the index of refraction of air (slide 13)
    4. Measure the index of refraction of a glass slide (slide 14)
    5. Measure interference patterns as a function of the relative polarization of the two interfering beams (slide 15)

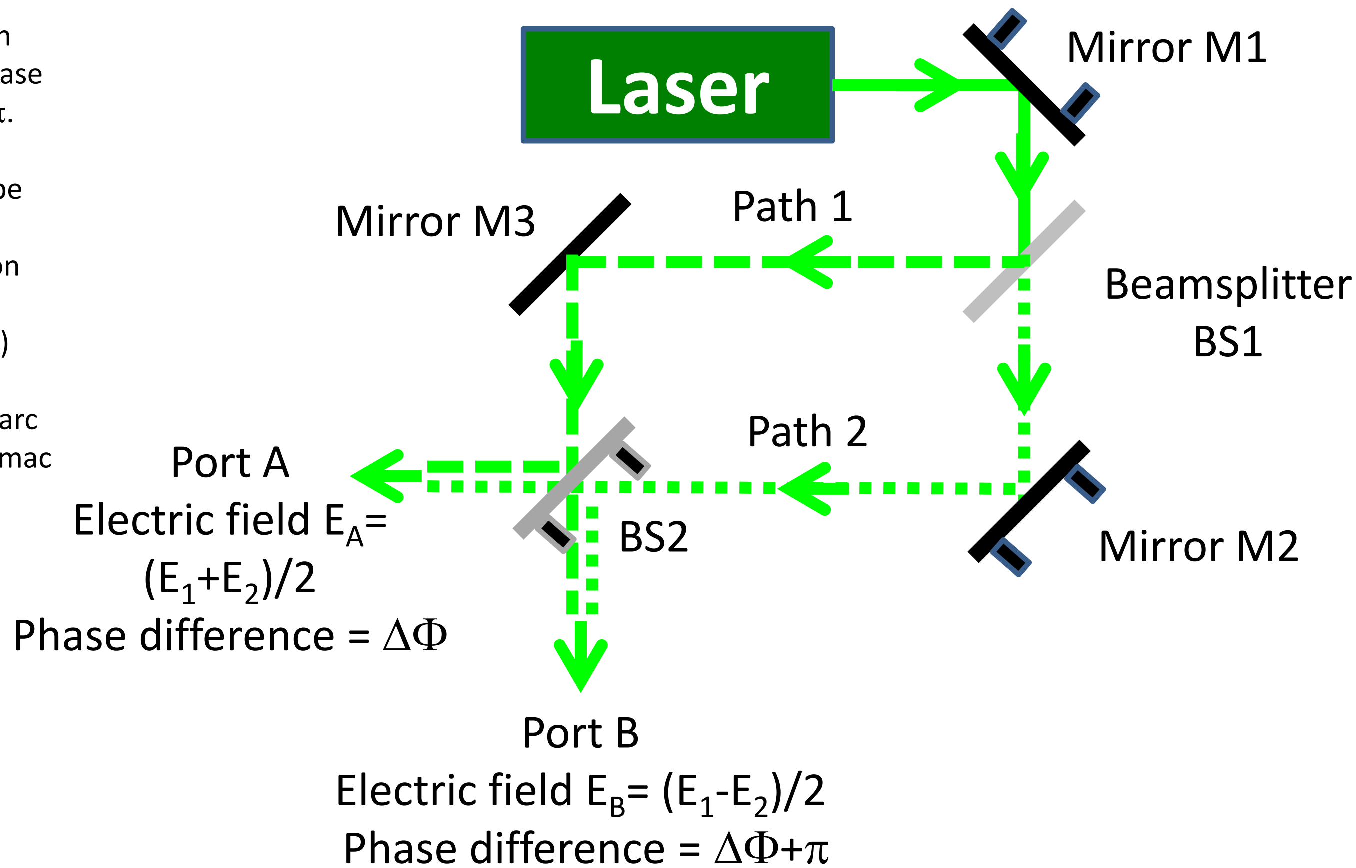
Lab 2: Mach-Zender Interferometer

Measurable is $\Delta\Phi = k_{path1} L_1 - k_{path2} L_2$

The interferometer measures $\Delta\Phi$ phase difference between two light beams, both of which originated from the same laser. Thus the two light beams have the same frequency and polarization. They also have the same intensity since they are created using 50% beamsplitters. In order to see interference easily, we have to combine the two beams so that they copropagate at both of the exit ports of the interferometer, as illustrated below.

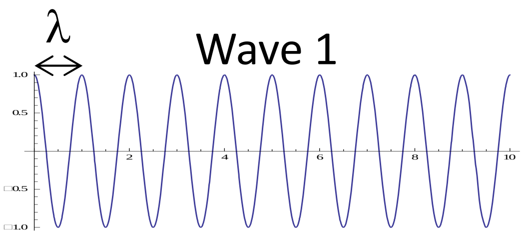
$\Delta\Phi$ is measured by the brightness or darkness of the light at port 1 and port 2. **The sum of the light from both ports is always equal to the initial intensity from the laser. What changes is how much of the light appears at each port.**

Let $\Delta\Phi$ be the phase difference between the two electric fields at port A. The phase difference at port 2 will always be $\Delta\Phi+\pi$. Thus, when interference at port A is constructive interference at port B will be destructive, and visa versa. The phase difference results from the phase shift on reflection. An article discussing this is posted on the website. Phys. Educ. 35(1) January 2000 https://www.cs.princeton.edu/courses/archive/fall06/cos576/papers/zetie_et_al_mach_zehnder00.pdf



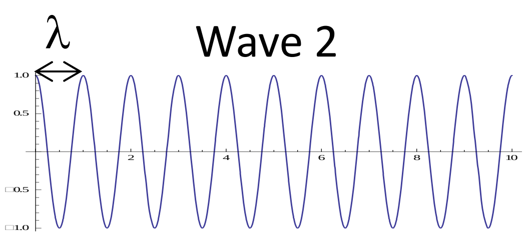
Observing Fringes due to Adding Waves

Consider two waves with exactly the same wavelength and amplitude that add together to form a new wave. **The only possible difference between the waves is their phase.**

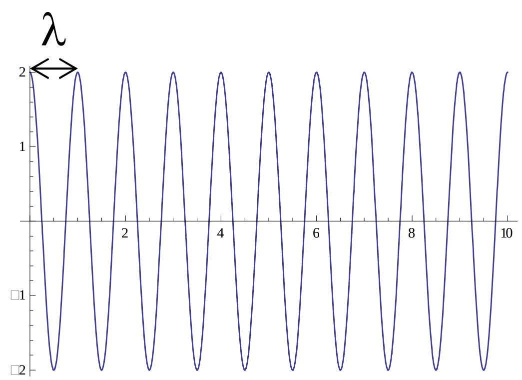


$E_1 = A\cos(kz - wt) = A\,\mathrm{Re}\,\mathrm{Exp}[-i(kz - w\,t)$

$K = 2\pi/\lambda$ where λ is the wavelength. It is the separation between adjacent amplitude peaks



$E_2 = A\cos(kz - wt + \Delta\Phi) = A\,\mathrm{Re}\,\mathrm{Exp}[-i(kz - w\,t + \Delta\Phi)$



In phase -> $\Delta\Phi = 0$

-> $E_1 + E_2 = 2A\cos(kz - wt)$

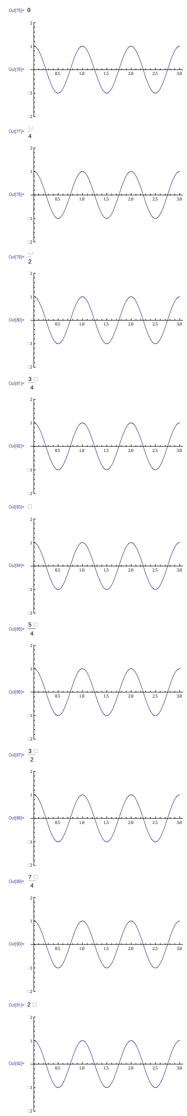
Pi phase shift -> $\Delta\Phi = \pi$

-> $E_1 + E_2 = A\cos(kz - wt) - A\cos(kz - wt) = 0$

General Phase shift

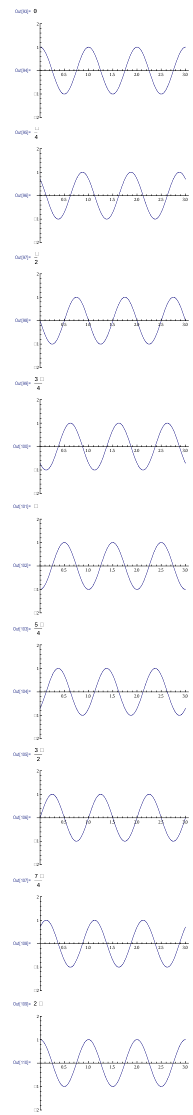
Wave 1

E_1= A cos (kz – wt)
= A Re Exp[-i(kz-w t)

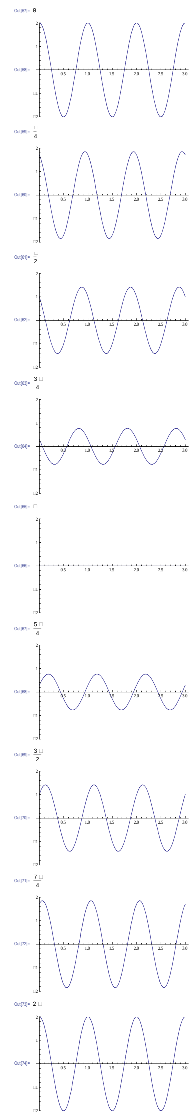


Wave 2

E_2= A cos (kz – wt+ $\Delta\Phi$)
= A Re Exp[-i(kz-w t + $\Delta\Phi$)



Sum



$\Delta\Phi$=0

$\Delta\Phi$=π/4

$\Delta\Phi$=π/2

$\Delta\Phi$=3π/4

$\Delta\Phi$=π

$\Delta\Phi$=5 π/4

$\Delta\Phi$=3π/2

$\Delta\Phi$=7π/4

$\Delta\Phi$=2π

Intensity is what we measure, not wave amplitude at a given time

Intensity =2 $A^2 \cos^2 (\Delta\Phi/2))$

We don't see electric fields, we see intensity. This is the time average of the square of the electric field.

In complex notation, the intensity is given by ½ the product of the e field and its complex conjugate

$E_{total}= E_1 + E_2 = A\ \text{Exp}[-i(kz-w\ t) + A\ \text{Exp}[-i(kz-w\ t+\Delta\Phi)$

-> Total Intensity

$= \frac{1}{2} (A\ \text{Exp}[-i(kz-w\ t) + A\ \text{Exp}[-i(kz-w\ t+\Delta\Phi)) (A\ \text{Exp}[i(kz-w\ t) + A\ \text{Exp}[i(kz-w\ t+\Delta\Phi))$

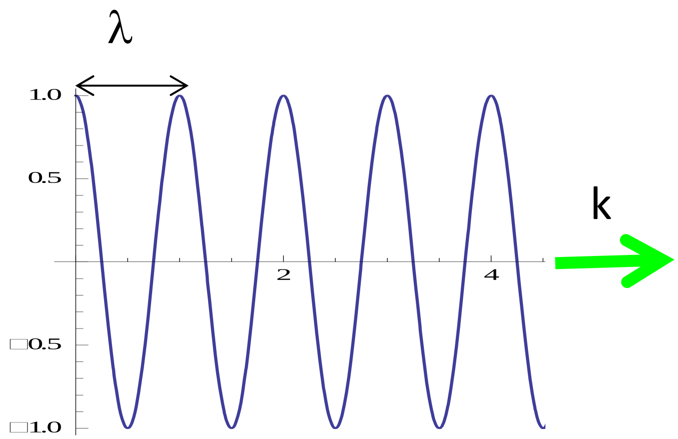
$= \frac{1}{2} A^2 (1+ \text{Exp}[i\ \Delta\Phi) + \text{Exp}[- i\ \Delta\Phi] + 1) = A^2 (1+ \cos (\Delta\Phi)) = 2 A^2 \cos^2 (\Delta\Phi/2))$

Test Special values

$\Delta\Phi=0$ -> $2A^2$, which is correct for adding two in phase fields with average intensity A

$\Delta\Phi=\pi$ -> 0, which is correct for adding two fields with opposite phases

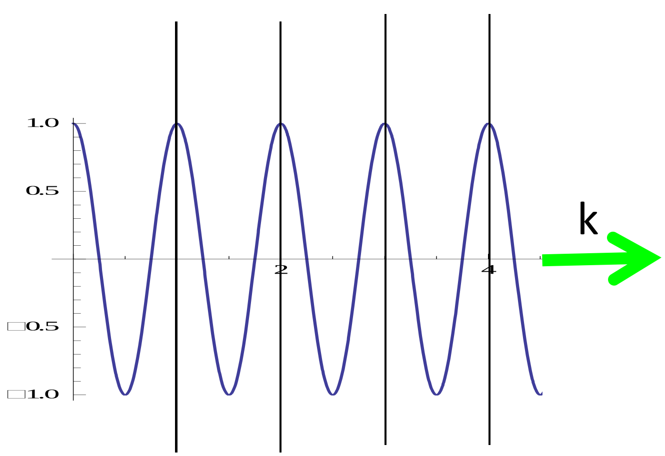
Laser Light is Composed of Plane waves



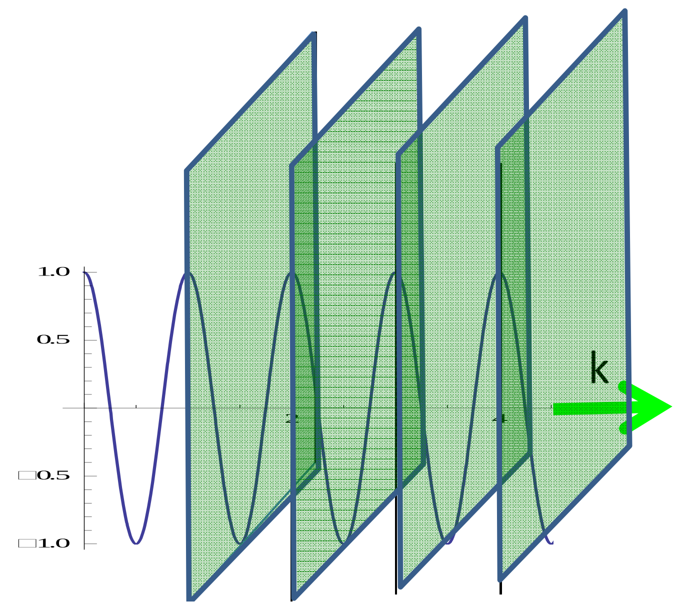
E_1= A cos (kz – wt)
= A Re Exp[-i(kz-w t)]

Light propagates in the z direction. The phase along the z direction varies as k z. k is the wavevector. It points in the direction of propagation (z). It has magnitude k = $2\pi/\lambda$. All positions with the same z value have the same phase. Thus, any plane with constant z is a plane of constant phase.

Each vertical line corresponds to a position occupied by an amplitude maximum. Each line is really part of a plane corresponding to maximum amplitude.

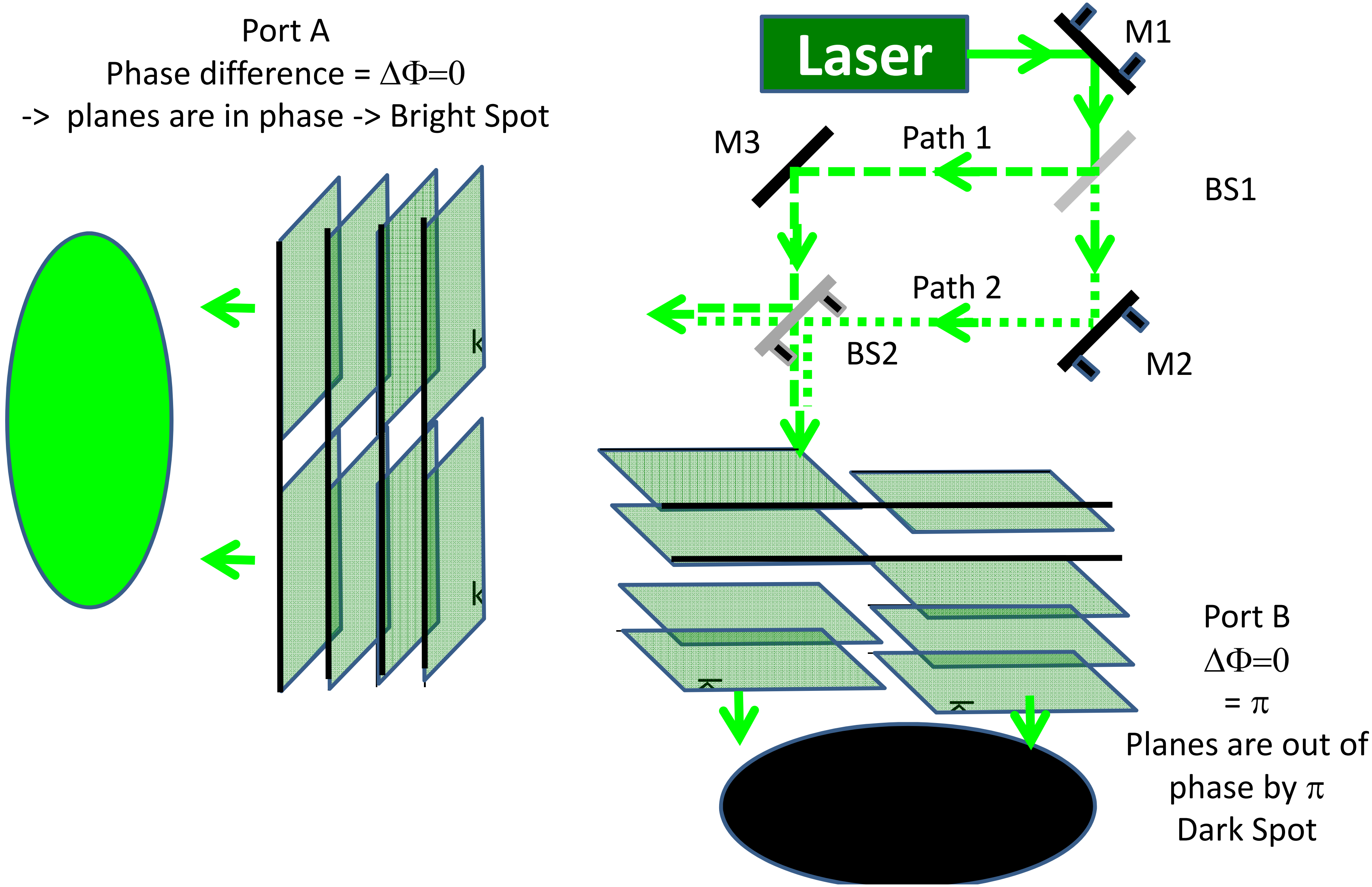


Planes corresponding to the four lines at left



Intensity at the ports of a Mach-Zender Interferometer

Measurable is $\Delta\Phi = k_{path1} L_1 - k_{path2} L_2 = k (L_1 - L_2)$



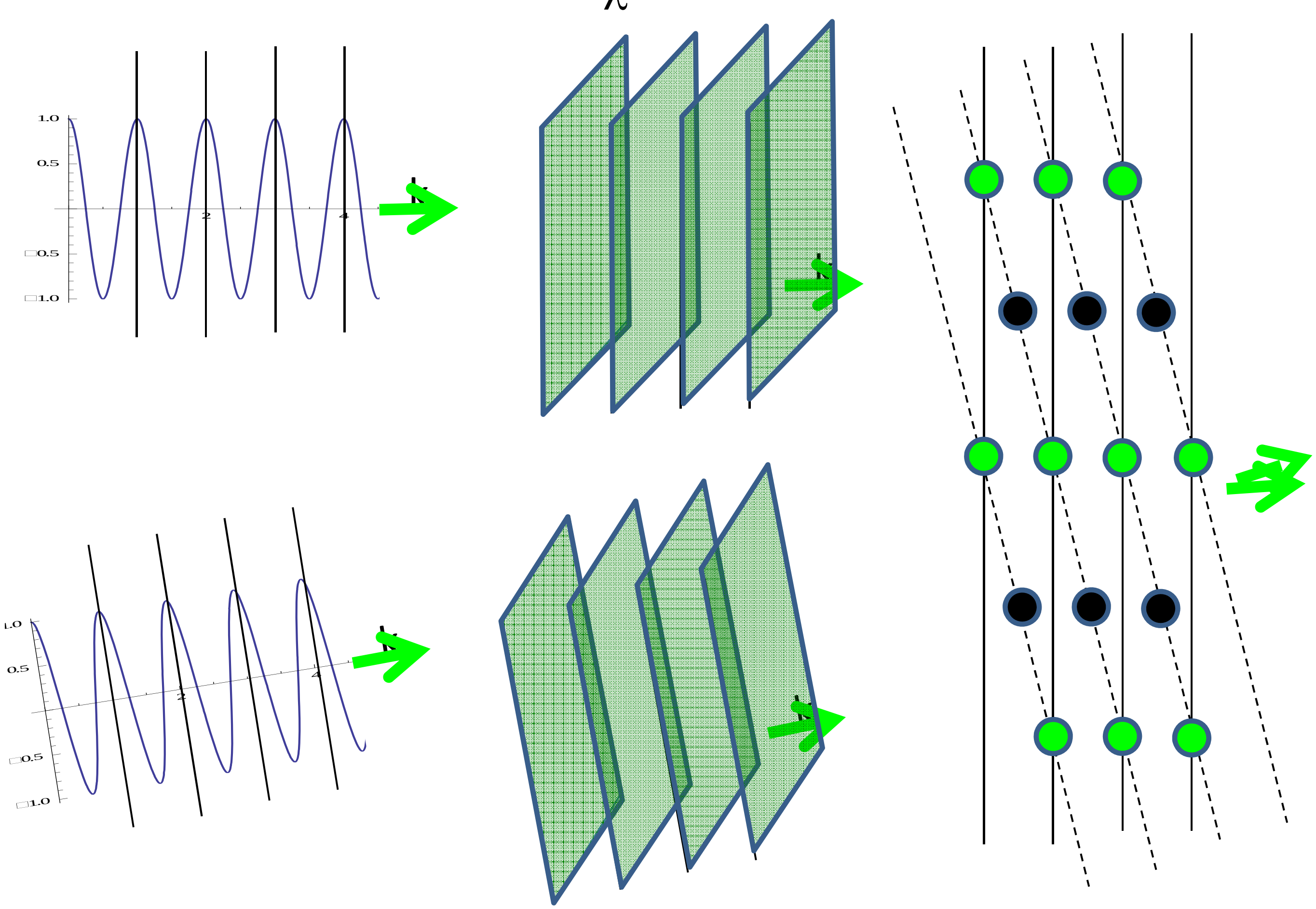
Slightly misaligned plane waves

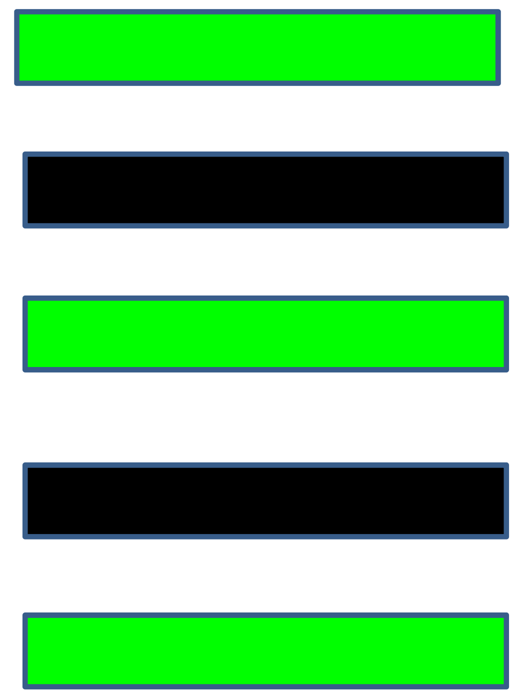
Measurable is $\Delta\Phi = k(L_1 - L_2)$. L_1 and L_2 vary with position across a screen at Port A or Port B. The variation is controlled by adjusting the mirror angles which control the angle between the propagation directions for the two beams.

Reduce planes to lines to see more clearly.
Dotted lines correspond to tilted waves.
Green circles show constructive interference points
Black circles show destructive interference

The line connecting the green circles corresponds to on phase positions. The line connecting the black circles corresponds to out of phase positions. The total intensity pattern is shown below. Vertical angular misalignment creates horizontal stripes

λ



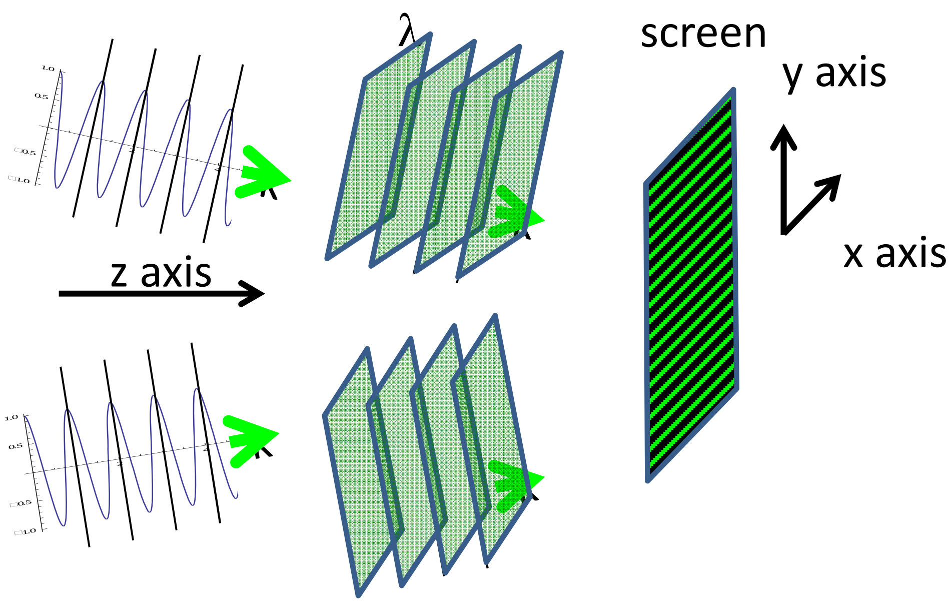


Slightly misaligned plane waves Math

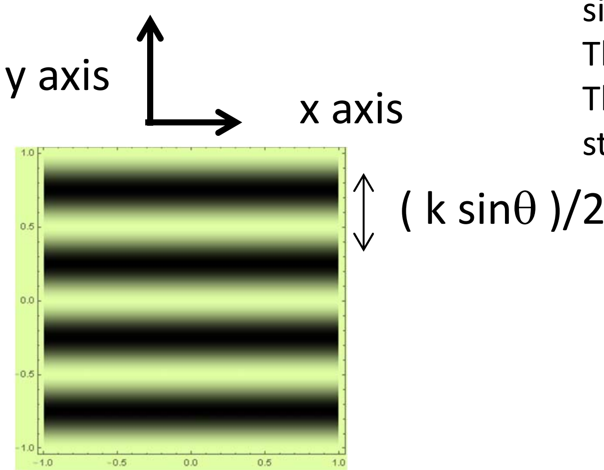
Intensity $=2 A^2 \cos^2 [k \sin\theta \ +\Delta\Phi/2)]$

Define the z axis as the direction that bisects the angle between the two propagation vectors

$K_1 = k \cos\theta\ z - k \sin\theta\ y$



$K_2 = k \cos\theta\ z + k \sin\theta\ y$



Viewing screen image looking along the z axis

$E_{total} = E_1 + E_2 = A\ \mathrm{Exp}[-i(k \cos\theta\ z - k \sin\theta\ y - w\ t)] + A\ \mathrm{Exp}[[-i(k \cos\theta\ z + k \sin\theta\ y - w\ t + \Delta\Phi)\ ]$

-> Total Intensity is 1/2 the product of the total E-field and its complex conjugate

$= \frac{1}{2} A^2\ (\ \mathrm{Exp}[-i(k \cos\theta\ z - k \sin\theta\ y - w\ t)] + \ \mathrm{Exp}[[-i(k \cos\theta\ z + k \sin\theta\ y - w\ t + \Delta\Phi)\ ])$

$(\ \mathrm{Exp}[i(k \cos\theta\ z - k \sin\theta\ y - w\ t)] + \ \mathrm{Exp}[[i(k \cos\theta\ z + k \sin\theta\ y - w\ t + \Delta\Phi)\ ])$

$= \frac{1}{2} A^2\ (1 + \mathrm{Exp}[i\ (2\ k \sin\theta\ y + \Delta\Phi)\ ] + \mathrm{Exp}[-i\ (2\ k \sin\theta\ y + \Delta\Phi)\ ] + 1)$

$= \ A^2 (\ 1 + \mathrm{Cos}[2\ k \sin\theta\ y + \Delta\Phi])$

$= 2\ A^2 \cos^2 [k \sin\theta\ y + \Delta\Phi/2)]$

Fringes along the y direciton have wavelength =(k sinθ)/2

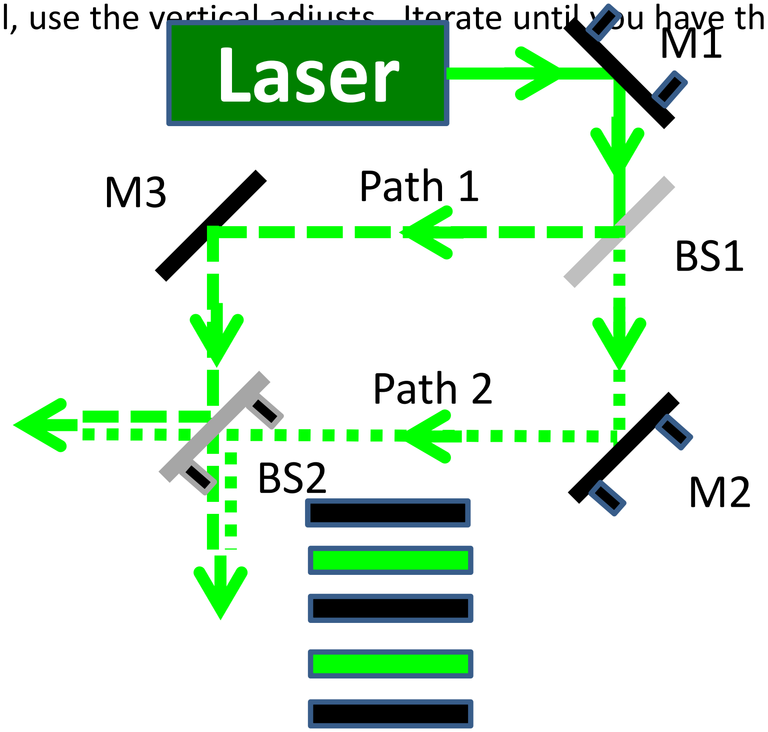
The pattern is invariant along the x direction.

Thus, vertical misalignment produces horizontal stripes

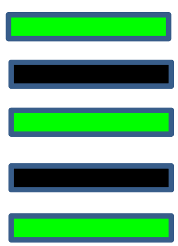
Deliverable 1

Measurable is $\Delta\Phi = k_{path1} L_1 - k_{path2} L_2 = k (L_1 - L_2)$

1. Show us the fringes in a Mach-Zender interferometer
   1. **Strategy :**
      1. Use mirror M2 to overlap both beams on the surface of BS2. Use BS2 to overlap beams at some distant point. Iterate until you see the beams overlapping over a long distance after BS2
      2. Open the aperture in front of the laser. You should see fringes. If the fringes are vertical, rotate the horizontal adjusts on mirror M2 and BS2 in the direction where the fringe size increases. If the fringes are horizontal, use the vertical adjusts. Iterate until you have the larges possible fringes.



Fringe patterns at ports A and Port B are opposite. Bright at port 1 corresponds to dark at port 2



Deliverable 2 Measure the distance change per volt for the piezo electric crystal

Measurable is $\Delta\Phi = k_{path1} L_1 - k_{path2} L_2 = k (L_1 - L_2)$ piezo changes only L_1

$\Delta\Delta\Phi = k \Delta L_1$. ΔL_1 is the change in length due to applying a voltage to the piezo

Strategy :

1. Replace mirror M3 with a mirror mounted on a piezo electric crystal whose length changes with applied voltage. Connect the wires from the piezo to a power supply. As you change the voltage, you should see the fringes change from bright to dark. If you have several stripes, the stripes will move.
2. Each time the interference pattern goes from bright to dark to bright, you have shifted the $\Delta\Phi$ by 2π. The only thing you are changing is L1. So the change in phase shift $\Delta\Delta\Phi = 2\pi N$ the number of fringes swept

$\Delta\Delta\Phi = k\Delta L_1$

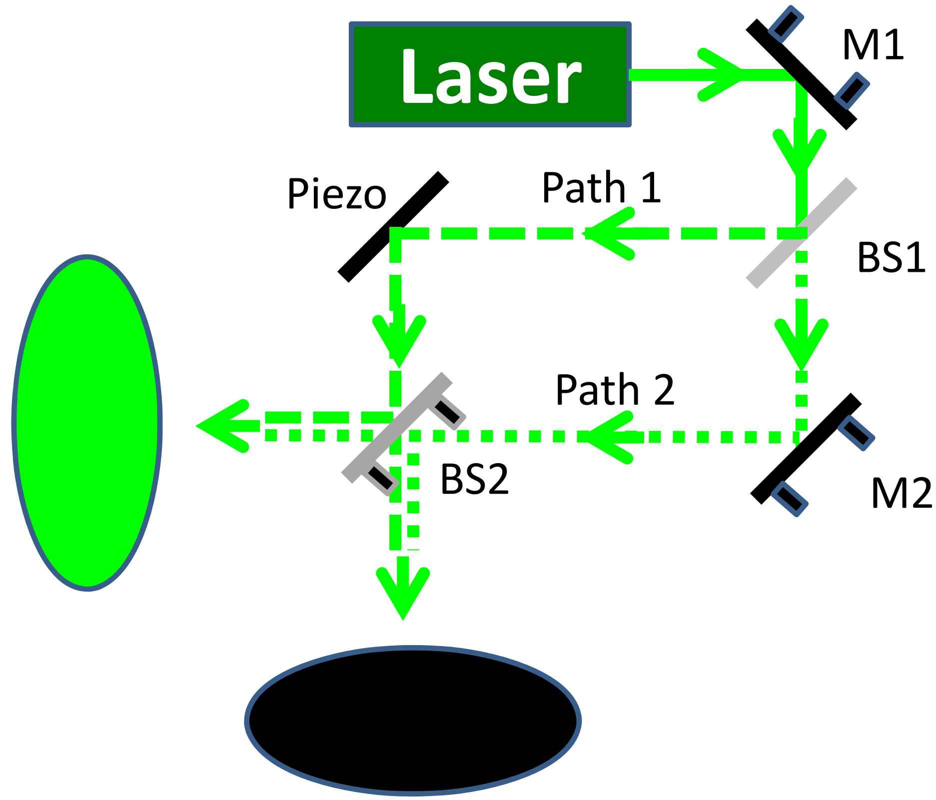
One choice is to choose the voltage change to correspond to exactly two fringe shifts. For this choice $\Delta\Delta\Phi = 4\pi$

This phase shift resulted from the length change due to the 2 volt voltage change so,

$\Delta\Delta\Phi = 4\pi = k\Delta L_1 = 2\pi/\lambda \, \Delta L_1$

Rearrange to give ΔL_1, the change in length due to sweeping through two fringes is

$2\lambda = \Delta L_1$. This makes intuitive sense, sweeping 2 fringes corresponds to a path change of 2λ. Now you have $\Delta L_1 = 2\lambda$ and your change in voltage, ΔV. You can now calculate $\Delta L/\Delta V = 2\lambda/\Delta V$



Deliverable 3 Measure the index of refraction of air

Evacuating the cell changes k1 INSIDE the cell, everywhere else path remains the same

$$\Delta\Delta\Phi = \Delta\Phi_{air} - \Delta\Phi_{vacuum} = \left[(k_{air} L_{cell} + k_{air} (L_1 - L_{cell}) - k_{air} L_2) \right] - \left[(k_{vacuum} L_{cell} + k_{air} (L_1 - L_{cell}) - k_{air} L_2) \right]$$

$$= L_{cell} \, (k_{air} - k_{vacuum})$$

Strategy :

1. Insert a glass cell into path. The cell begins filled with air.
2. Evacuate the cell and count the number of fringe shifts as you are evacuating the cell. Each time the interference pattern goes from bright to dark to bright, you have shifted the $\Delta\Phi$ by 2π. The only thing you are changing is L1. So the change in phase shift DDF

If the wavelength in a vacuum is λ_o, the wavelength in a material with index of refraction n is λ_o/n

$\Delta\Delta\Phi = k_{air} L_{cell} - k_{vacuum} L_{cell}$

$= L_{cell} \, (2\pi/(\lambda_o/n) - 2\pi/\lambda_o)$

$= L_{cell} \, 2\pi/\lambda_o (n-1)$

You counted N fringes, so the phase shift you measured is 2π N.

Equate your measure and calculated phase shifts

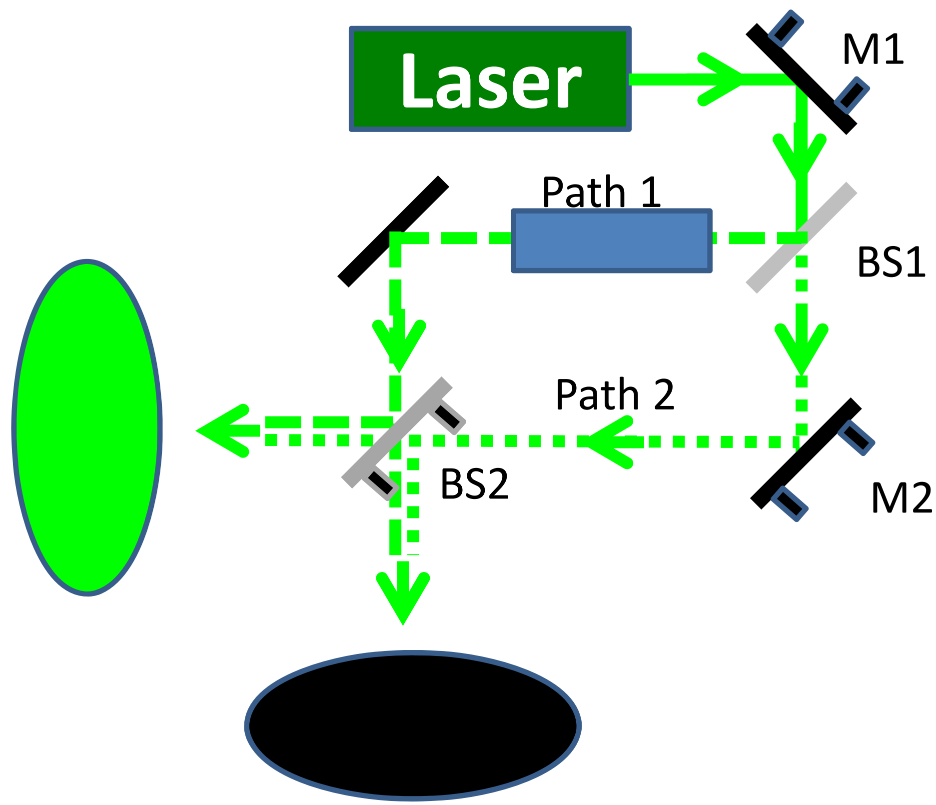
$2\pi N = 2\pi L_{cell} / (n-1)$

-> $(n-1) = N \lambda_o / L_{cell}$

For N=twenty fringes

-> (n-1)= 20 ($.5 \times 10^{-6} / 0.06$)~ 1.6 x 10-4

-> n for air ~ 1.00016



- **Deliverable 4: Measure the index of refraction of glass**
- Measurable is $\Delta\Phi = k_{path1} L_1 - k_{path2} L_2$ Path 2 is unchanged, but path 1 has changes in L1 and k_{path1} . Calculate changes in air and glass paths then add.

1. **Strategy :**
   1. Rotate microscope slide to change the path length traveled through air and glass. Count, N, the number of fringes created by the rotation as you measure θ_{air}. You need to use Snell's law to get θ_{glass}. $\sin\theta_{glass} = \sin\theta_{air}/n_{glass}$
   2. Measured phase shift $\Delta\Delta\Phi_{meas} = N (2\pi)$
   3. Calculated phase shift = $\Delta\Delta\Phi_{calc} = k_{air}\,\Delta D_{air} + k_{glass}\,\Delta L_{glass}$
      1. $K_{air} \sim 2\pi n_{air}/\lambda o$ $k_{glass} = 2\pi/\lambda_{glass} = 2\pi\, n_{glass}/\lambda_o$
   4. $\Delta\Delta\Phi_{calc} = 2\pi/\lambda_o\,(n_{air}\ T\,(1-\cos(\theta_{air} - \theta_{glass})\,/(\cos\theta_{glass}))$
      $+\ n_{glass} T\,(1/\cos\theta_{glass} - 1)$
   5. Equate measured and calculated $\Delta\Delta\Phi$
   6. $\Delta\Delta\Phi_{meas} = N (2\pi) = \Delta\phi_{calc}$

Taylor series expansion to second order in θ_{air} gives
$n_{glass} \sim$
$(\theta_{air}^2 T/\lambda_o)/(\theta_{air}^2\ T/\lambda_o - 2N\ )$
$= 1/(1 - 2N\lambda_o/(\theta_{air}^2\ T\ )$

Air Path Length Change

When glass is perpendicular to the light, $L_{air} = L_1 - T$. As the glass tilts, $L_{air} = L_1 - D$.

$D = L\cos\alpha = L\cos(\theta_{air} - \theta_{glass})$

$\Delta L_{air} = (L_1 - D) - (L_1 - T) = T - D$

$= T\,(1 - \cos(\theta_{air} - \theta_{glass})\,/(\cos\theta_{glass})) < 0$

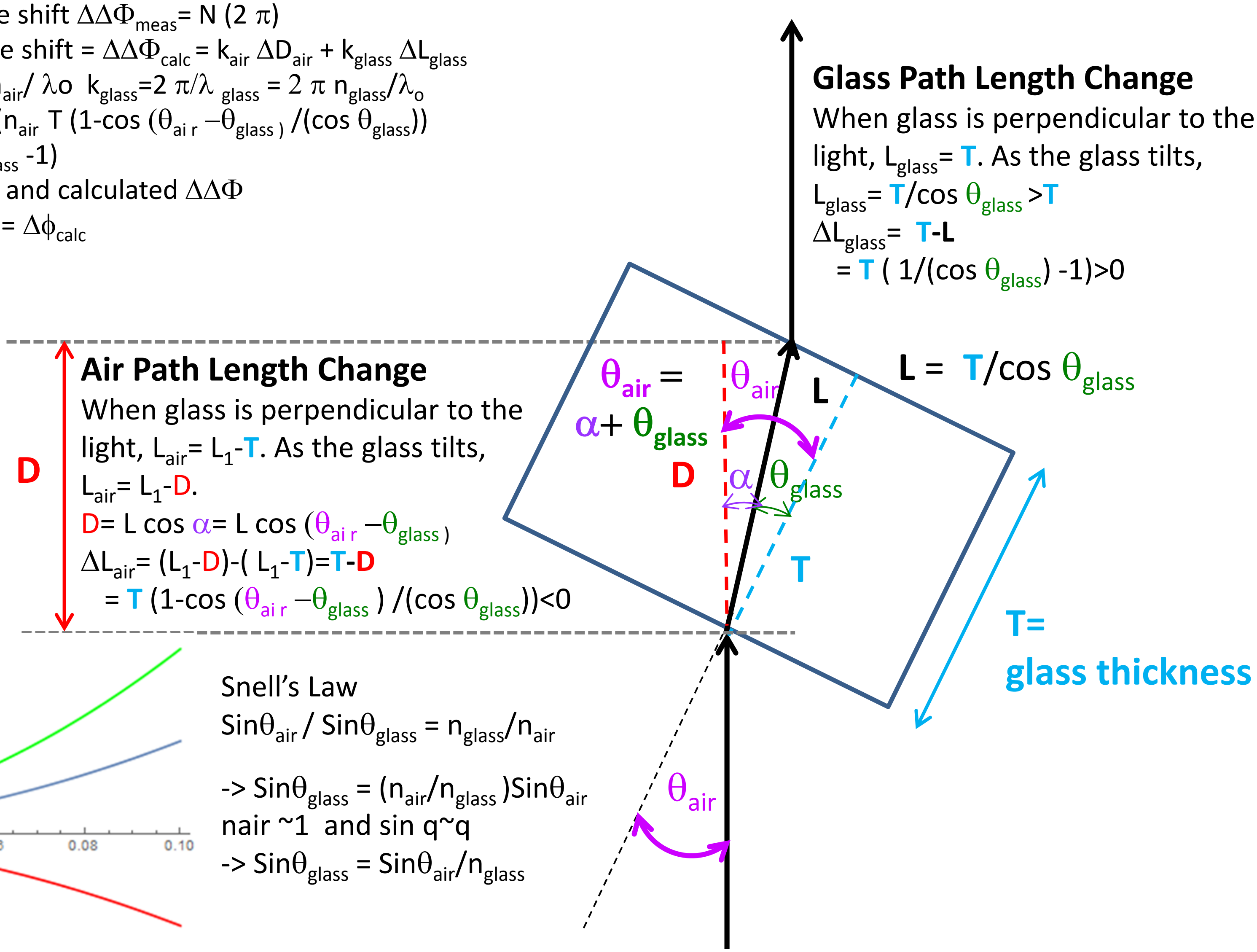
Glass Path Length Change

When glass is perpendicular to the light, $L_{glass} = T$. As the glass tilts,

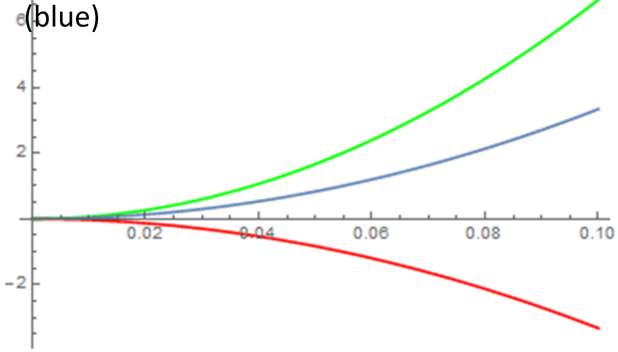
$L_{glass} = T/\cos\theta_{glass} > T$

$\Delta L_{glass} = T - L$

$= T\,(1/(\cos\theta_{glass}) - 1) > 0$



For n_{glass}=1.5 plots of for $\Delta\Phi/(2\pi)$ vs θ_{air} for lass (green), air (red), and total (blue)



Snell's Law

$\sin\theta_{air} / \sin\theta_{glass} = n_{glass}/n_{air}$

-> $\sin\theta_{glass} = (n_{air}/n_{glass})\sin\theta_{air}$

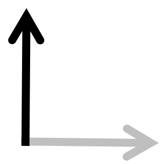
nair ~1 and sin q~q

-> $\sin\theta_{glass} = \sin\theta_{air}/n_{glass}$

- **Deliverable 5: Measure the influence of polarization on interference**
- Measurable 1: Influence of polarizer on E-field vector for light
- **Strategy :**
  1. Use linear polarizers to measure and control the light polarization
  2. Show the laser is linearly polarized by inserting a linear polarizer in the laser beam and rotating the polarizer. The light that passes through the polarizer is polarized along the pass axis of the polarizer. The intensity transmitted is proportional to the cos^{2}q, where q is the angle between the polarization vector of the laser light and the pass axis of the polarizer.

Test 1 look at laser light transmitted through a linear polarizer

Laser Polarization direction = x

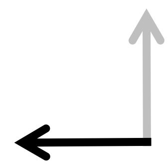


Insert polarizer 1 into the laser beam. The black axis represents the polarization direction that is blocked by the polarizer, and the gray arrow represents the pass axis.

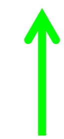
No Light Transmitted through the polarizer

Test 2 look at laser light transmitted through a linear polarizer

Laser Polarization direction = x

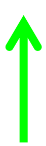


Rotate polarizer 1 by 90 degrees

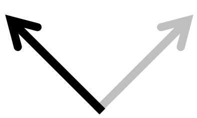


All Light transmitted through the polarizer

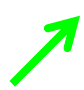
Test 3 look at laser light transmitted through a linear polarizer



Laser Polarization direction = x



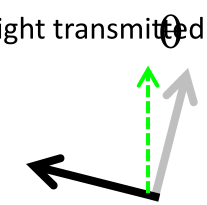
Rotate polarizer 1 by 45 degrees

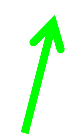


Amplitude = Cos($\pi/4$)
Intensity =1/2

Test 4 look at laser light transmitted through a linear polarizer

Laser Polarization direction = x



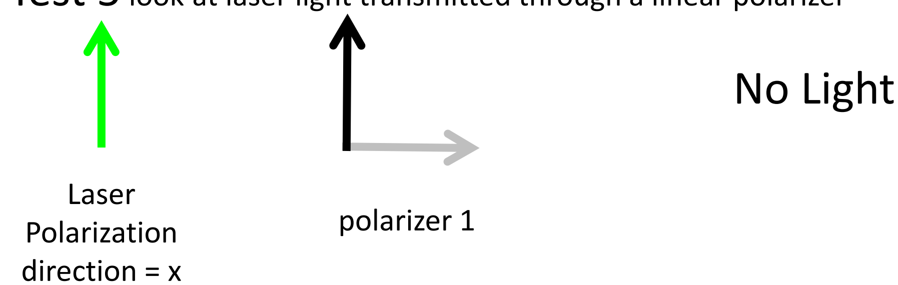


E field Amplitude = Cos(θ)
Intensity = Cos^2(θ)

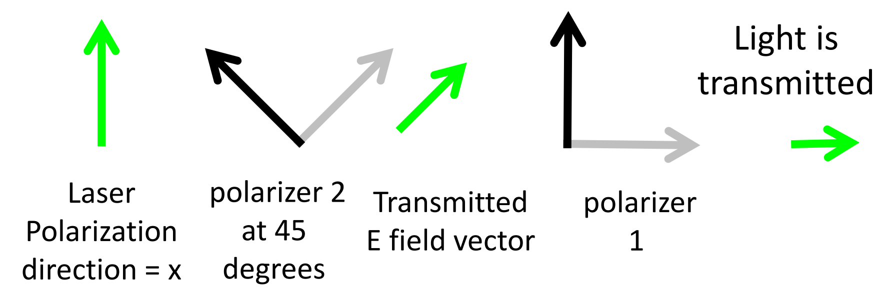
Light vector after passing through the polarizer lies along the pass axis. Its amplitude is reduced by cosθ, the angle between the pass axis and the laser polarization. The intensity transmitted is proportional to cos$^2\theta$

- **Deliverable 5: Measure the influence of polarization on interference**
- Measurable 1: Influence of polarizer on E-field vector for light
- **Strategy :**
  1. Use linear polarizers to measure and control the light polarization
  2. Show the laser is linearly polarized by inserting a linear polarizer in the laser beam and rotating the polarizer. The light that passes through the polarizer is polarized along the pass axis of the polarizer. The intensity transmitted is proportional to the $\cos^2 q$, where q is the angle between the polarization vector of the laser light and the pass axis of the polarizer.

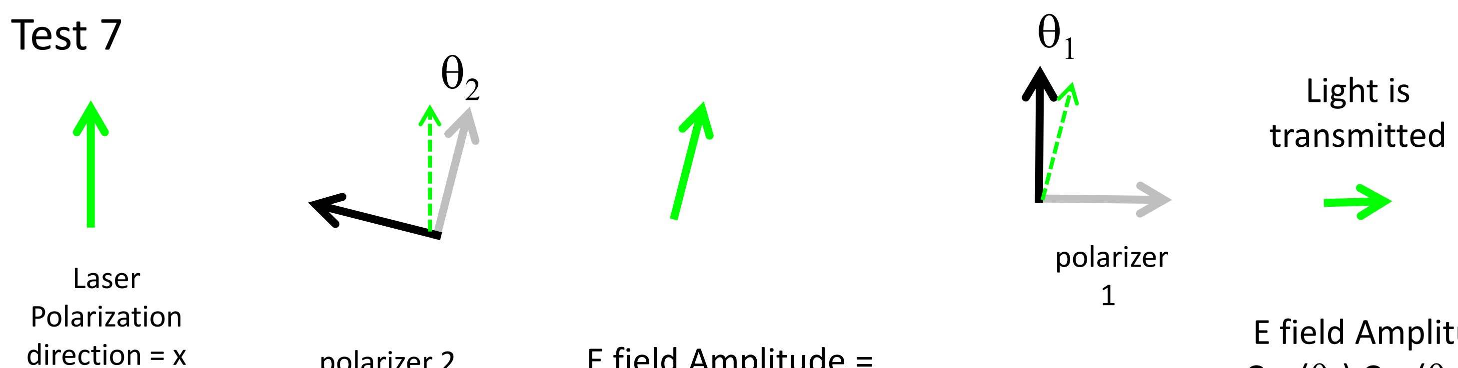
Test 5 look at laser light transmitted through a linear polarizer



Test 6 add polarizer 2 between the laser and polarizer 1 and study the effect on the light transmitted by polarizer 1



Test 7



Laser
Polarization
direction = x

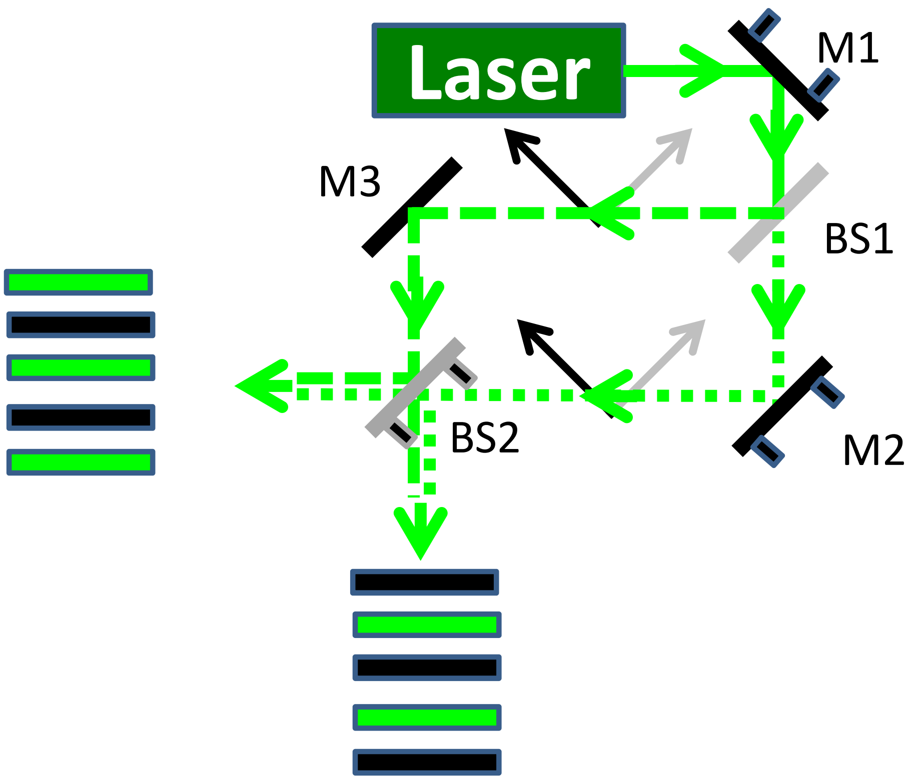
E field Amplitude = $\cos(\theta_2)$
Intensity = $\cos^2(\theta_2)$

E field Amplitude = $\cos(\theta_2)\cos(\theta_1)$

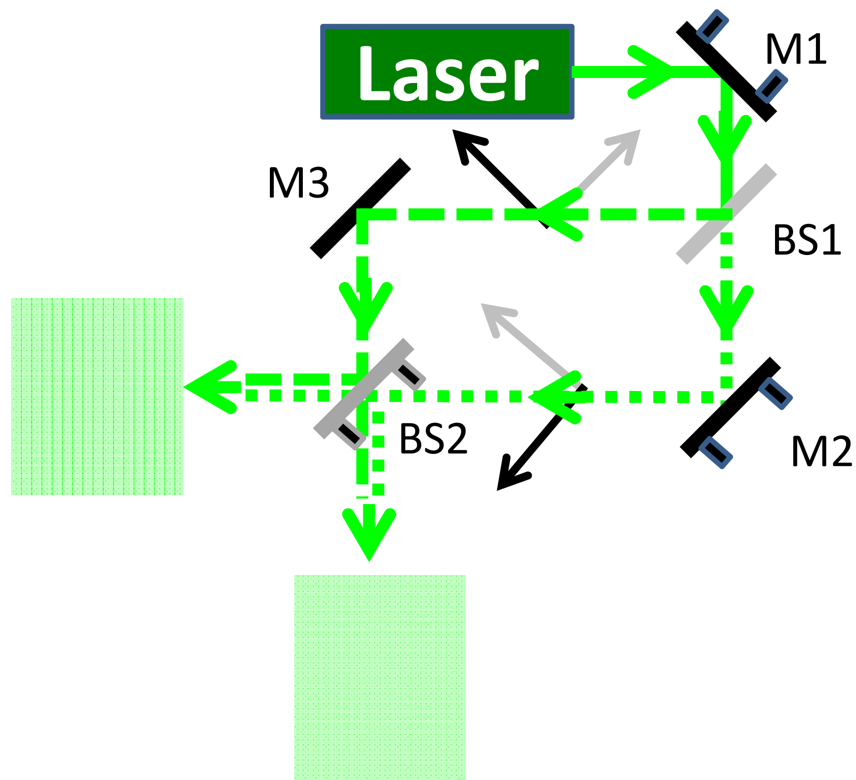
Intensity = $[\cos(\theta_2)\cos(\theta_1)]^2$

- **Deliverable 5: Measure the influence of polarization on interference**
- Measurable 2: Measure the influence of polarization on the output of the Mach Zender.
- **Strategy :**
  1. Use linear polarizers to measure and control the light polarization separately within each arm of the interferometer
  2. If the E-field vectors from both paths are the same, the interference is unchanged by the polarizers
  3. If the E-field vectors from both paths are the perpendicular, there is no interference pattern

Same polarization -> same fringes as without polarizers



Perpendicular polarization -> no fringes because the vector dot product of the E-field vectors is zero



The End