Harmonic Oscillation

- **Learning Goals**

After you finish this lab, you will be able to:

1. Describe (quantitatively and qualitatively) the motion of a real harmonic oscillator

2. Find a mathematical function that fits the motion of an oscillator.

3. Describe and predict the motion of a damped oscillator under different damping conditions.

Introduction: Please read all of this BEFORE you come to lab.

Background and introduction

- In this lab, you'll explore the oscillations of a mass-spring system, with and without damping. You'll see how changing various parameters like the spring constant, the mass, or the amplitude affects the oscillation of the system. You'll also see what the effects of damping are and explore the three regimes of oscillatory systems—under-damped, critically damped, and over-damped.

Harmonic motion

Most of what you need to know about harmonic motion has been covered in the lectures, so we won't repeat it in depth here. The basic idea is that simple harmonic motion follows an equation for sinusoidal oscillations:

$$x_{\text{undamped}} = A\cos(\omega t + \phi)$$

We have added here a *phase* ϕ, which simply allows us to choose any arbitrary time as $t = 0$. (Otherwise, using a cosine function, we would have to choose $t = 0$ when the mass was at its maximum displacement from the origin.)

For a mass-spring system, the angular frequency, ω, is given by

$$\omega = \sqrt{\frac{k}{m}}$$

where m is the mass and k is the spring constant. Note that ω does not depend on the amplitude of the harmonic motion.

Damping

The situation changes when we add *damping*. Damping is the presence of a drag force or friction force, which is non-conservative; it gradually removes mechanical energy from the system by doing negative work. As a result, the sinusoidal oscillation does not go on forever. Mathematically, the presence of the damping term in the differential equation for $x(t)$ changes the form of the solution so that it is no longer a simple sine wave.

A damped system will be in one of three regimes, depending on the amount of damping. Small damping causes only a slight change in the behavior of the system: the oscillation frequency decreases somewhat, and the amplitude gradually decays away over time according to an exponential function. This is called an *under-damped* system. The solution to the equation of under-damped systems is:

$$x_{\text{underdamped}} = Ae^{-\gamma t}\cos(\omega_{\text{d}} t + \phi)$$

The parameter γ indicates how rapidly the oscillation decays and the parameter ω_{d} is the new (damped) oscillation frequency:

$$\gamma = \frac{b}{2m}$$

$$\omega_{\text{d}} = \sqrt{\omega^2 - \gamma^2}$$

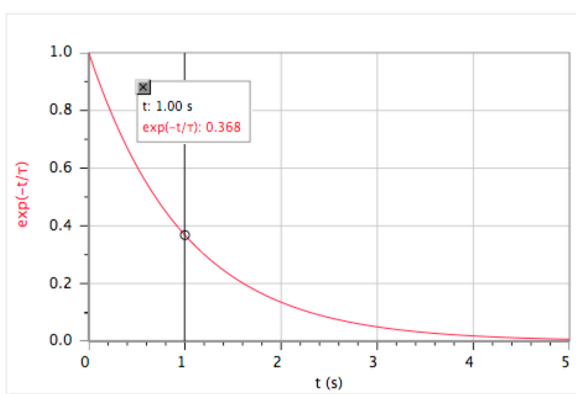
Here b is the damping constant. So for small damping (low b), the decay is very slow (small γ), and ω_{d} is only slightly less than the un-damped angular frequency ω.

The "Underdamped" fit function in Logger Pro is a cosine wave whose amplitude decreases exponentially with time: position = A*exp(-t/B)*cos(C*t+D)+E. Note that the fit parameter B is equal to $1/\gamma$.

The key difference between this and un-damped motion is the exponential factor, exp(–t/B). The fit parameter B is called the *time constant* of the motion; it represents how quickly the amplitude decreases. The time constant is usually denoted with the Greek letter tau (τ), but Logger Pro doesn't do Greek letters in curve fitting, so that's why it's B in the equation.

Quantitatively speaking, τ is equal to the amount of time it takes for the amplitude to decrease by a factor of $1/e$, where e is the constant 2.718, the base of the natural logarithm. ($1/e$ is about 0.37.)

At right is a graph of exp($-t/\tau$) for τ = 1.00 second:



If the damping is very large, the system does not oscillate at all. In fact, if it is displaced from equilibrium, it takes a long time for it to even return to its initial position because the drag force is so severe. Such a system is called *over-damped*. It turns out that this behavior is very nearly described by a simple exponential function:

$$x_{\text{overdamped}} = Ae^{-\gamma t}$$

Here, the parameter γ depends on the damping coefficient b **in a different way** than for under-damped systems. For over-damped systems, γ is always less than ω, the angular frequency of un-damped oscillation.

Exactly at the transition between over-damping and under-damping is a regime known as *critical damping*. As with over-damping, a critically damped system does not oscillate, but it returns to equilibrium faster than an over-damped system. It also follows (approximately) the negative exponential, but with a larger value of γ, which allows it to return to equilibrium faster than an over-damped system.

In fact, this is the defining characteristic of critically damped systems: **they return to equilibrium quickly and stay there**. An over-damped system is slow to return to equilibrium because it's just slow, period. An under-damped system gets to equilibrium quickly, but overshoots it and keeps oscillating about it, albeit with a gradually diminishing amplitude. This is the reason critical damping is interesting: in many applications (e.g. shock absorbers), you'd like any oscillations to damp out as quickly as possible.

Helpful Logger Pro tip: Making a graph using data on page 2

If you want to make a graph but the data you want to plot do not appear on the list of variables, check to see if the variables that do appear are all on the same page and the ones that do not appear are on a different page. If this is the case, you can make the data from the different page appear by changing the variable in the x-axis.

- Double-click on the graph you want to change, or go to "Options" on the menu bar and scroll down to "Graph Options"
- A new window will appear.
- Select the tab "Axes Options"
- At the bottom left, select which variable you want on the x-axis from the drop-down list next to the label "Column"
- The variables that appear on the y-axis column will be the variables associated with the variable you selected for the x-axis column.
- When you are done, click "OK"

Experiment 5: Lab Activity

- Follow along in this lab activity. Wherever you see a question highlighted in red, be sure to answer that question, or paste in some data, or a graph, or whatever is being asked. Your lab report will be incomplete if any of these questions remains unanswered.

Who are you? Take a picture of your lab group with Photo Both and paste it below along with your names.

- **Materials:**

  - 3 springs: Type A, long (thin); Type A, short (thin); Type B, short (thick)
  - Mass stand and masses

    The mass stand is a hook with a tray at the bottom for putting masses on it. The stand itself has a mass of 50 grams, the CD attached to it has a mass of 20 grams, and there should be a 100 g mass attached. There is also an additional 50 g mass.

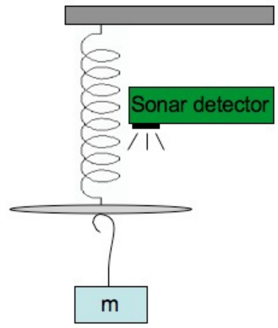
  - Lab jack
  - Sonar motion detector

    The sonar motion detector is a sensor that detects the position of objects using sonar ranging. The **minimum distance** away from the sonar detector that objects can be "seen" is 15 cm (about 6 inches). The resolution of the detector is 0.3 mm (that is, an object has to move by at least 0.3 mm in order for the sonar detector to read a different position measurement for it). The sampling frequency of this motion detector is about 30Hz.

  - 600 mL plastic beaker
  - 25 mL graduated cylinder
  - Plastic water bottle
  - Karo corn syrup (consult a TF before using)
  - Stirring rod
  - Forceps
  - Plastic tray
  - Computer with Logger Pro

Harmonic Oscillation

Part 1: (Nearly) Undamped oscillations



In this part of the lab, you will determine the angular frequency of a 170-g mass (50 g for the stand, 20 g for the CD, and a 100 g mass) oscillating on a long (not very stiff) spring. Hang the mass-spring system high over your lab bench and place the sonar detector above it. The CD on the hanging mass is so that the detector can "see" the motion of the hanging mass.

Remember, the sonar detector can only detect objects a *minimum distance of 15 cm away*. Also, use only small amplitudes of oscillation; this will significantly decrease the chances of having the mass fall off the stand and break something and it gives cleaner data.

Zero the motion sensor, then pull the mass down and let go. Record data for the motion of the hanging mass for several periods, but **only a few seconds** (about 5 cycles). Try to get everything lined up vertically and minimize side-to-side motion.

After you have taken some data, use it to estimate the period of the oscillation by inspecting the position vs. time graph.

What is the period of oscillation?

Period =

Calculate the angular frequency from the period that you measured.

ω =

Try fitting a sinusoidal curve to the position vs. time graph. Under "General Equation," scroll down to the option near the bottom called "Undamped." The general equation for un-damped motion is position = A*cos(C*t+D)+E.

Which parameter in the fit corresponds to angular frequency?

What do the other parameters correspond to?

What is the angular frequency from the fit?

ω =

How does this compare to the angular frequency from estimating the period?

Which method for determining the angular frequency is more accurate? Why?

Paste a copy of the position vs. time graph, including the sinusoidal fit, below:

From your data and the sinusoidal fit, calculate the spring constant of the spring (don't forget units):

***k* =**

Qualitatively, predict how *ω* will change if you have a larger mass oscillating on the same spring?

Add 50 g of extra mass to the mass stand, pull down, and let go.

What is the angular frequency of the resulting motion?

ω =

How does this measured value compare to your prediction above?

Pull down on the mass stand again, with larger amplitude than before, and let go.

What is the angular frequency of the resulting motion?

ω =

Is it the same, larger, or smaller than the angular frequency of the motion with smaller amplitude?

Now hang the mass stand with the extra mass to the stiffer spring.

Predict (qualitatively) how will the angular frequency will change compared to the case where you had a spring that was not as stiff.

Pull down on the mass stand and let go.

What is the angular frequency of the resulting motion?

ω =

How does this value compare with your prediction in the previous question?

Part 2: Damping due to air drag

For this part of lab, you will use a total mass of about 170 g and a type A, long spring oscillating with amplitude of a few centimeters. Pull down on the mass, let go, and observe the motion of the mass for **two minutes**. (Change the data collection settings before taking data, if you need to. Instructions on how to do this are in the Logger Pro help file.)

Describe what you observe.

Is the motion under-damped or over-damped?

Fit the data to a curve based on your observation of the motion.

Paste a copy of your position vs. time graph, including the fit parameters, here:

What is the angular frequency of the oscillatory motion?

ω =

What is the time constant for the motion?

τ =

Looking at the graph, compare the amplitude of the motion at the beginning, and at a time B seconds later.

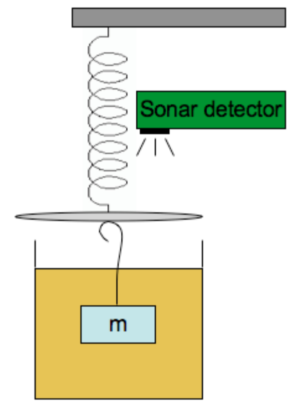
What factor do they differ by?

What would happen to the time constant τ (or fit parameter B) as you change the amount of damping?

Part 3: Damping due to viscous drag

In this part of the lab, you will explore damping due to viscous drag by observing the motion of an oscillating mass that has been submerged in corn syrup. Please refer to the supplemental material for notes on setting up the experiment. The main question you will be answering is: **At what dilution of water-syrup is the system critically damped?** You will begin with pure, 100% corn syrup. Then you will add 40 mL of water at a time, take data of the motion of the hanging mass, and extract some parameters from your data (which you will use to fill the table below). This isn't exactly a titration, so you don't need to be as careful as you would in a chemistry lab.

To set up the experiment for damping with viscous drag, pour between 325–350 mL of corn syrup into the beaker and set it on the lab jack. Position the mass over the beaker and raise the height of the jack until the mass is resting in the center of the beaker as shown on the picture at left. You can also adjust the height of the bar holding the spring.



Make sure the mass is fully submerged. If it is too far down, the motion will be hindered by the bottom of the beaker; if it is too close to the surface, the surface tension of the syrup will hinder the motion.

Wait until the system is fully at equilibrium.

NOTES: Corn syrup is very viscous. You have to be patient and careful with things moving in corn syrup. Diluting water in corn syrup takes time (see previous note). Be careful and make sure you don't lose any fluid and that the water and syrup are well mixed before taking more data. Hot water works better than cold or warm water.

Make sure you zero the detector before taking data.

Record all your data in the following table:

Dilution (mL)	Time constant (s)	Time to zero (s)	Angular Frequency (rad/s)
0			
40			
80			
120			

Set up the experiment as described above. Start with only corn syrup in the beaker. While collecting data, and without touching the spring directly, pull the mass up so that the top of the mass is level with the top of the syrup and then release it. After it has come to a complete stop, you can stop the data collection.

Describe what you observe.

How does this relate to what you know about damped harmonic motion? Is the motion over- damped or under-damped?

Fit the data to a curve based on your observation.

What is the time constant of the motion?

τ =

From visually inspecting the graph, estimate when the mass reaches a position within 5 mm of its final equilibrium position. Subtract the starting time (the time when you released the mass) to get the time elapsed until the mass reached equilibrium.

What is this time?

Time to equilibrium =

Save your data run by pressing command-L.

On page 2 of the Logger Pro file you will see a data table like the one above. Enter your time constant in the top row, in the column labeled τ. Enter your time to equilibrium in the last column, labeled "Time to zero." The third column (angular frequency) for this case will be empty.

Why is there no angular frequency for this case?

Remove the mass from the corn syrup without changing the height of the jack. Add 40 mL of water, mix well, and repeat the experiment to determine the time to equilibrium and the angular frequency (if applicable). Take data for dilutions of 0 mL, 40 mL, 80 mL, and 120 mL of water, being sure to mix thoroughly for each dilution. Make sure you save each run (**command-L**) and fill the table above and on page 2 of the Logger Pro file.

Describe your observations as you dilute the corn syrup:

What does τ represent if the system is under-damped?

For under-damped motion, “time to zero” can refer to the time it takes the system to come to equilibrium, or to the time when the system first crosses the equilibrium point. Both definitions are OK; for consistency let’s choose the time when **the system first crosses the equilibrium point**.

Critical damping is the threshold behavior between over-and under-damped motion. From your data on page 2,

Which dilution comes closest to critical damping?

What was the time constant when the system was closest to critical damping?

τ =

What was the time to zero for the system closest to critical damping?

Time to zero =

Paste a copy of the position vs. time graph for the system when it was closest to critical damping:

Create a graph showing both τ and Time to zero vs. dilution and paste it here. If you are having trouble finding the right variables, check out the helpful tips from the Introduction above.

In words, what conclusions can you draw from this graph regarding the relationship among τ, Time to zero, and the regimes of damping?

Look at the values of ω that you observed for the under-damped system(s).

How does ω for the under-damped liquid system(s) compare to the ω you got for the same system damped only by air drag?

For under-damped systems, does ω increase, decrease, or stay the same as the amount of damping goes up?

- When you have finished, ***clean up anything in your work area that has corn syrup all over it*** (the beaker, mass stand, masses, stirring rod, forceps if you used them, and anything else which was in the splash radius of your corn syrup) ***by taking it to the sink and rinsing it out thoroughly with warm water. Leave everything to dry on the tray ON YOUR LAB BENCH (not at the sink).***

- **Conclusion**

What is the most important thing you learned in lab today?

What aspect of the lab was the most confusing to you today?