15c Fourier Optics Lab
(Exercises to be Submitted are listed at the End)

1. **Review of geometric optics**
   a. Missions:
      1. Image the ceiling fluorescent lights onto the floor using a lens
      2. You two colored LED's that are next to each other. Turn on the LED's and image them using only an aperture (Try it in one of the dark rooms at the side of the lab). Image the same LED's using the lens instead of the pinhole.
      3. Draw ray optics explanations of what you did in the 2 exercises. Give two reasons why cameras with lenses are preferred to pinhole cameras, and provide one advantage of a pinhole camera
   b. Possible resources **NOT** required
      1. http://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html
      2. http://spie.org/Documents/Publications/00%20STEP%20Module%2003.pdf
      3. http://astrowww.phys.uvic.ca/~tatum/goptics/geomop2.pdf

2. **Wave Optics Introduction**
   a. Missions
      1. Draw the wave fronts and k vectors corresponding to a plane wave propagating along the z direction
      2. Draw the wave fronts and k vectors corresponding to a diverging spherical wave
      3. Draw the wave fronts and k vectors corresponding to a converging spherical wave
      4. Write the mathematical expression for all of the waves above given that they propagate with velocity c, so the period $T=\lambda/c$, since the wave travels one wavelength during one period. By definition $T=2\pi/\omega$ and $k=2\pi/\lambda$
      5. **Huygen's wavelet theory states that the forward propagation of a wave can be replicated by point sources if the point sources are placed along the positions occupied by a wavefront from the original wave (surface of constant phase)**
      6. Go to http://www.falstad.com/ripple/ and use it to do the following. If you right click on a point source you can duplicate it. This provides you with another point source with the same phase.
         1. Create a propagating spherical wave from a point source
            a. Explain how the image relates to the wave fronts and k vectors in ii
         2. Combine 2 point source of the same frequency, change the separation between the sources and explain what you are seeing
         3. Combine many sources to create a plane wave
         4. Combine many sources to create a converging spherical wave

5. Create a source that mimics a single very narrow slit and explain results
6. Create a source that mimics a single wide slit, or explain which previous exercise already achieved this mission.

7. Create a source that mimics two narrow slits and explain results or explain which previous exercise already achieved this mission.

7. Explain how what you did is related to Huygen's theorem.

b. Possible resources **NOT** required
1. http://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle
2. http://farside.ph.utexas.edu/teaching/302l/lectures/node150.html
3. https://www.boundless.com/physics/wave-optics/diffraction/huygens-principle/
4. Very math intense http://mathpages.com/home/kmath242/kmath242.htm

3. **Lenses and focusing**
   a. Missions
      1. Given a spherical diverging point source at z= -R , calculate the phase shift as a function of (x,y) in the z=0 plane that would convert the diverging spherical wave into a plane wave propagating along the z direction
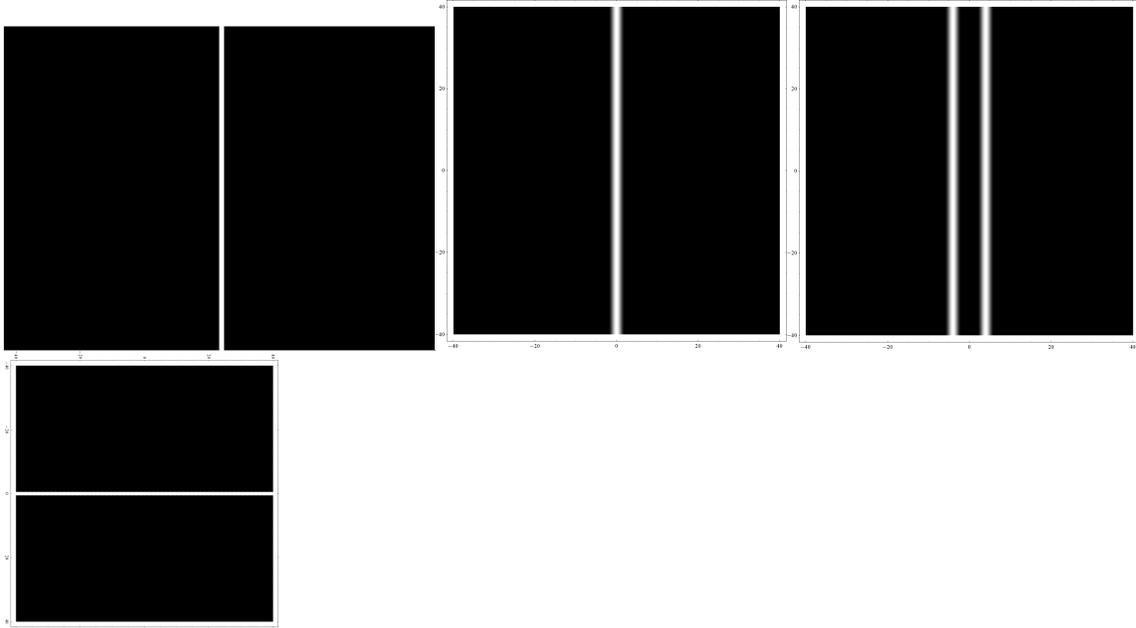   b. Possible resources NOT required
      1. http://www.owlnet.rice.edu/~dodds/Files332/fourier.pdf
      2. http://en.wikipedia.org/wiki/Fourier_optics
      3. http://www.optics.unm.edu/sbahae/Optics%20Lab/Fourier%20Optics.pdf

4. **Fourier Transform Optics of Slits. The images at the end of this section provide the slits when appropriately positioned on the computer screen.**
   a. **Note a light source at position z=0 that passes through a lens at z=f (f=focal length) will produce an image that is the Fourier transform of the object at position z=2f. Adding another identical lens at z=3f will reproduce both the phase and the amplitude of the original source at position z=4f.**
   b. Review Fourier series if needed: http://mathworld.wolfram.com/FourierSeries.html
   c. Review Fourier transforms if needed: http://en.wikipedia.org/wiki/Fourier_transform
   d. 15b flashback
      1. Low pass filter on a square wave gave a sine wave
      2. High pass filter highlights edges, corresponding to the derivative
      3. Marilyn Monroe and Albert Einstein

4. Go to http://www.falstad.com/fourier/ look at square wave with 1 odd order, 2 odd orders, 3 odd orders. Go to all orders

e. **Mission Overview** using the 4F setup to look at the x and y components of the k vectors that are generated by diffraction by g(x,y), the amplitude variation created by the LCD make. Explore how filtering the x and y components changes the image by blocking part of the light in the Fourier transform plane. If you close down the aperture, you block k vectors with large components in the x and y direction If you use the needle, you block the small k vectors with small components in the x and y directions. The original plane wave only has a k vector in the z direction, so it will appear as a single spot at the origin since it corresponds to kx=ky=0.

f. **Missions**
   1. Show mathematically that a light source at position z=0 that passes through a lens at z=f (f=focal length) will produce an image that is the Fourier transform of the object at position z=2f. Adding another identical lens at z=3f will reproduce both the phase and the amplitude of the original source at position z=4f
   2. Look at diffraction due to a single vertical slit with square edges. Note: you can use the zoom control on the image to continuously tune the effective size of the slit
      a. Close the aperture in the Fourier transform plane and look at the image of the grating. How does it change? Explain
      b. Open the aperture, but place the needles so that it blocks the central spot in the Fourier transform plane and look at the image of the grating. How does it change? Explain
      c. Optional: look at the diffraction due to a single horizontal slit with square edges
   3. Look at diffraction due to a single slit with smooth edges
      a. Close the aperture in the Fourier transform plane and look at the image of the grating. How does it change? Explain
      b. Open the aperture, but place the needles so that it blocks the central spot in the Fourier Transform plane and look at the image of the grating. How does it change? Explain
   4. Double slit with smooth edges. How does this result compare with what you got in part 7 of the previous section where you used the applet to create two slit interference?

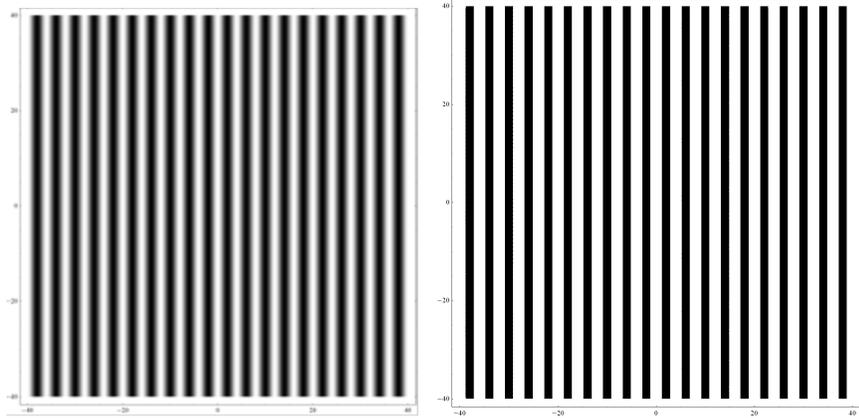g. Possible resources **NOT** required
   1. http://scienceworld.wolfram.com/physics/FraunhoferDiffraction.html
   2. http://en.wikipedia.org/wiki/Fraunhofer_diffraction
   3. http://en.wikipedia.org/wiki/Kirchhoff's_diffraction_formula
   4. http://optics.hanyang.ac.kr/~shsong/13-Fresnel%20diffraction.pdf
   5. http://www2.ph.ed.ac.uk/~wjh/teaching/mo/slides/scalar/scalar.pdf
   6. http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/fraunhofcon.html
   7. http://lpc1.clpccd.cc.ca.us/lpc/molander/pdfs/diffraction.pdf
   8.

5. Fourier transform optics beyond slits. The first 3 missions are mandatory
   a. Missions

   2. Periodic Gratings
      1. Below are diffraction gratings with smooth edges and square edges. Note you can change the effective period by zooming in and out on the image
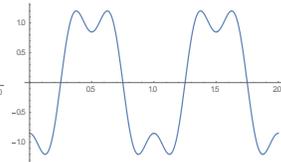
Note a square wave can be Fourier decomposed into sine waves
http://mathworld.wolfram.com/FourierSeriesSquareWave.html

$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right).$$
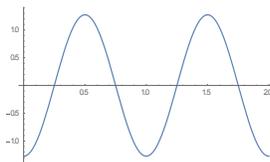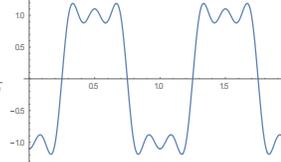
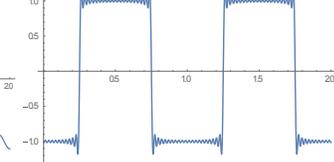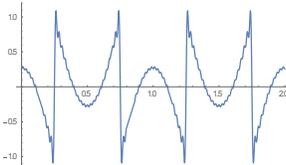First order only   Two orders only   Three orders only   Twenty orders only

Twenty orders without the first order

2. Use the needle or the aperture to double the frequency of the square wave diffraction grating. Explain

3. Convert a square wave to a sine wave without changing the grating by using the needle or the aperture. Explain

2. A grating of squares

Look at the diffraction pattern due to the following square gratin
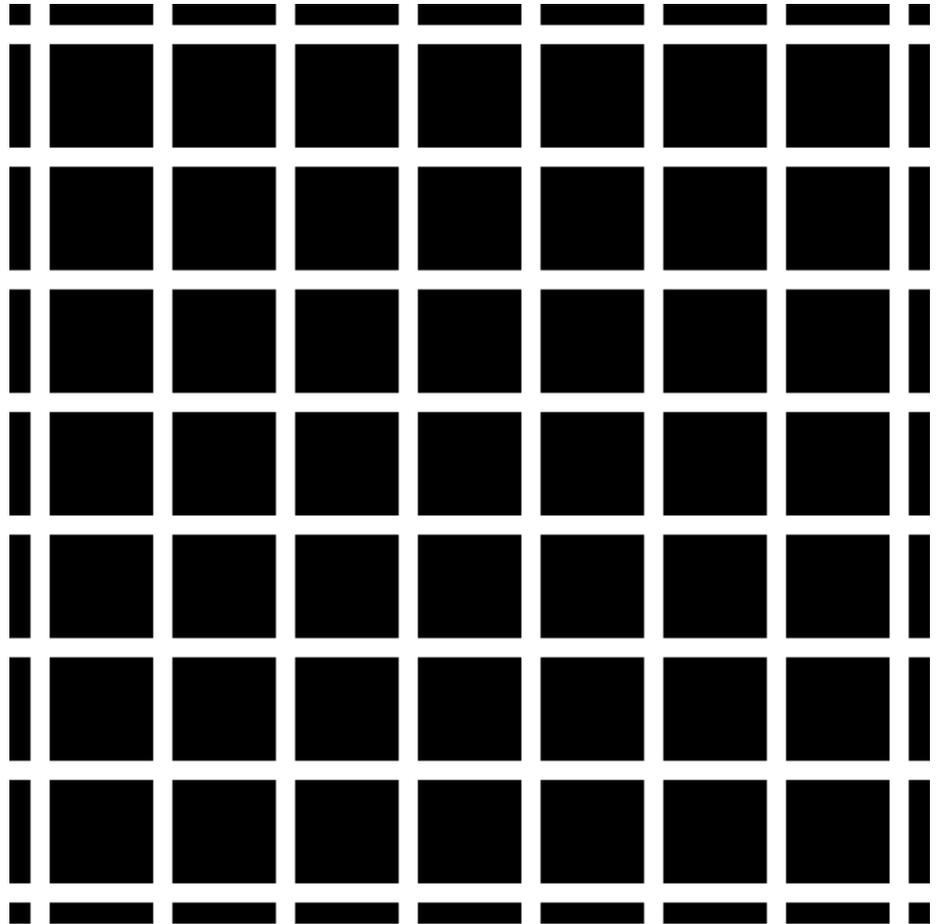
1. Now use two pieces of tape to create a slit in the Fourier plane, allowing only one direction of the diffraction pattern to pass. Predict the image you see and test your prediction

3. **Introduction to holograms:** Calculate the optical transmission mask required to convert a plane wave into a spherical wave and compare your results to the images shown below. Which ones matches your calculation?

   Look at the diffraction pattern due to the images below. Move the images around on the screen and look at the resulting motion of the diffraction pattern. Explain differences in the results for the 4 images. **Important: Study the intensity pattern by look directly a piece of paper inserted between the LCD and the aperture.** Scan the piece of paper back and forth and look at the intensity pattern.
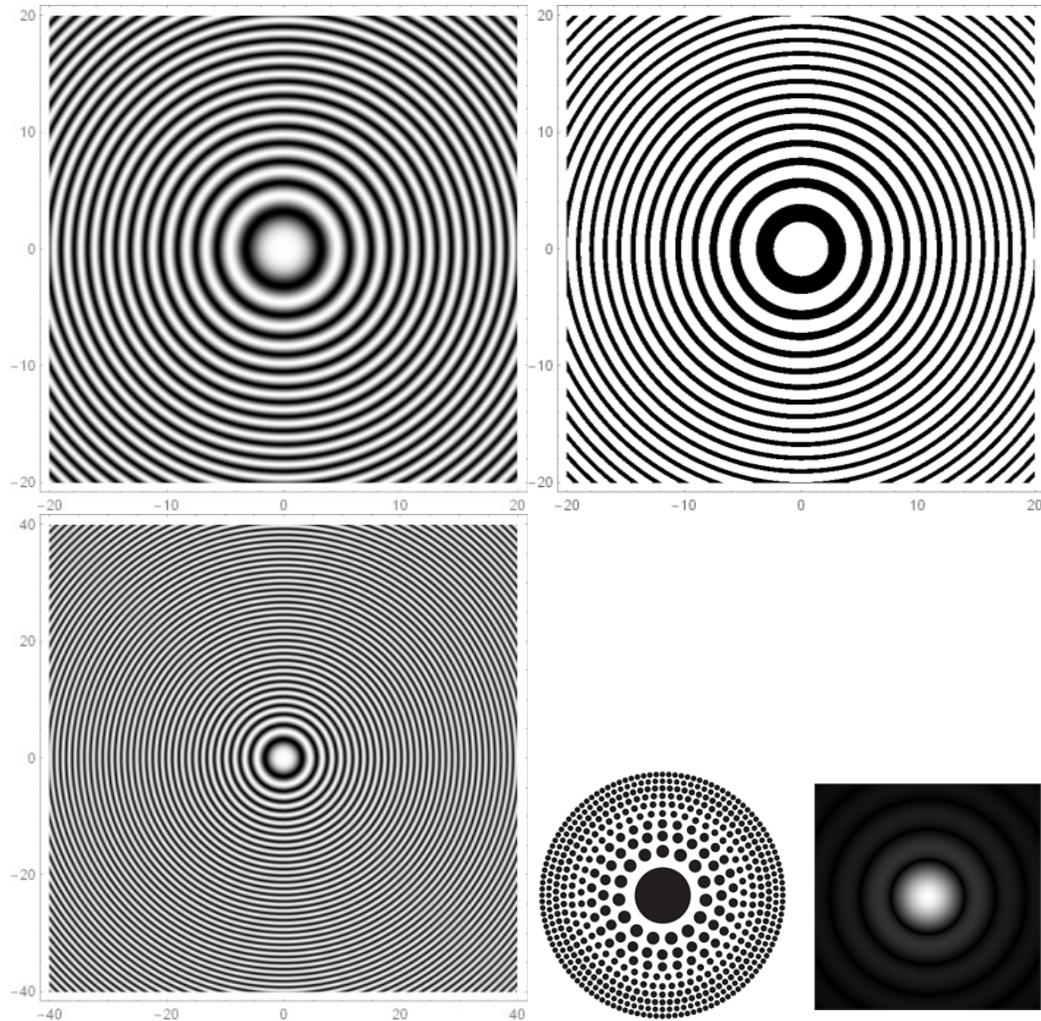
4. Optional possibilities

5. Make a diffraction pattern with Exp[- alpha x^2] and explain the diffraction pattern for various alpha
6. Look at the diffraction pattern for a point source
7. Make the queen mother look young again

8.
9. Convert Marilyn Monroe into Albert Einstein
   http://www.123opticalillusions.com/pages/albert-einstein-
   marilyn-monroe.php

10.
11. Try anything you want (try highpassing the Boston skyline)

Appendix on Controlling the LCD Display and Data Analysis Software

LCD Control
The liquid crystal display should automatically match the computer display pixel-by-pixel.
 Matlab program to create bars is 'image_bars01.m'.
    N=number of bars
    W=width in pixels
    Spacing = period in pixels.
Run program.
View file '15c.jpg'. One way to do this is to control-click in the matlab file list, and choose "open outside of matlab". It will open in Preview.

Data Analysis Control

 Matlab program to integrate camera output is 'photobooth_vert_integrate.m'. Take a picture in Photobooth, run the Matlab program, and click the button on the GUI to load the most recent. It is possible to choose a limited vertical region for the integration by entering a pixel range into the GUI. Note that larger numbers correspond to lower position in the image.

Exercises to be Submitted

1. Draw ray optics explanations of imaging due to a lens and imaging due to a pinhole camera. Explain what you did in the 2 exercises. Give two reasons why cameras with lenses are preferred to pinhole cameras, and provide one advantage of a pinhole camera.
2. Draw the wave fronts and k vectors corresponding to a plane wave propagating along the z direction
3. Draw the wave fronts and k vectors corresponding to a diverging spherical wave
4. Draw the wave fronts and k vectors corresponding to a converging spherical wave
5. Write the mathematical expression for all of the waves above given that they propagate with velocity c.
6. Describe how you used the Falstad applet to construct a plane wave and a spherical wave using point sources and explain how this relates to Huygen's theorem.
7. Given a spherical diverging point source at $z= -R$, calculate the phase shift as a function of (x,y) in the $z=0$ plane that would convert the diverging spherical wave into a plane wave propagating along the z direction
8. Explain how the electric field amplitude $E(x'',y'')$ at aperture in the 4f system relates to the Fourier transform of $E(x,y)$ the electric field amplitude at position (x,y) in the LCD screen. Provide both a ray optics and wave optics explanation of the role of the lens.
9. For the exercise where you looked at diffraction due to a single vertical slit with square edges explain the changes you observed in the image of the slit when you closed the aperture and when you inserted the needle to block the central spot in the fourier transform plane. Compare the diffraction pattern for the square slit with the diffraction pattern for the smooth slit and explain any differences
10. How does this diffraction pattern for the double slit compare with what you got in part 7 of the previous section where you used the applet to create two slit interference?
11. How did you reduce the wrinkles in the queen mother's face?
12. For the square wave diffraction grating how did you double the frequency you saw in the 4f plane and how did you produce a sine wave grating in the 4f plane?
13. When you looked at the diffraction pattern due to the squares, what happened when you used tape to create a slit in the Fourier transform plane? Explain your results.
14. Provide an equation for the optical transmission mask required to convert a plane wave into a spherical wave. Demonstrate that you can change the focal length of your zone plate and explain how you did it. Bonus: explain how you would design a zone plate that has a dark circle at the center. **Note you have created a simple hologram. Next week's lab will look at more complex holograms**