Experiment 1: The Same or Not The Same?

• Learning Goals

After you finish this lab, you will be able to:

1. Use Logger Pro to collect data and calculate statistics (mean and standard deviation).
2. Explain the difference between standard deviation and standard error.
3. Calculate a 95% confidence interval (CI) for the mean of a set of measurements.
4. Use the confidence interval to determine whether two measurements are the same, or not the same.

• Introduction: Please read all of this BEFORE you come to lab.

Your doctor tells you that your cholesterol level is 200, and you should try to get it down. She advises you to improve your diet and get some more exercise. After three months, you go back for another cholesterol test. Your new cholesterol level is 190.

So: it dropped from 200 to 190. Did diet and exercise make a difference?

The answer is: you don’t know, unless you are told the uncertainty of these cholesterol measurements. What do we mean by uncertainty?

Let’s say that the cholesterol test is accurate to within ±10 points. That would mean that a reading of 200 could actually represent 200 ± 10. The new reading of 190 could actually represent 190 ± 10. In other words:

Old reading of 200 means: your actual cholesterol level is between 190 and 210
New reading of 190 means: your actual cholesterol level is between 180 and 200

Since these intervals overlap, you can’t conclude that your cholesterol has changed. In fact, it is possible that your actual cholesterol level increased!

So: If the test is only accurate to ±10 points, there is essentially NO difference between readings of 200 and 190. Your cholesterol might have gone up, or down, or remained the same, and you can’t tell.

On the other hand, if the test is accurate to within ±2 points, you have:

Old reading of 200 means: your actual cholesterol level is between 198 and 202
New reading of 190 means: your actual cholesterol level is between 188 and 192

Now, since these intervals don’t overlap, you can be confident that your cholesterol level actually did decrease.

The first lesson we want you to take away from this lab is: you can’t compare two measurements unless you know something about the uncertainty of those measurements. Now we need to learn a bit more about uncertainty…
* Now would be a good time to read the **Introduction to Error Analysis by Taylor** that is posted on the website.

**Gaussians**

You know from reading the Taylor excerpt that all measurements are subject to some random variation, which we refer to as “uncertainty”. Much of the time, this random variation is distributed according to a “bell curve” like the following:

![Gaussian Distribution](image)

This function has lots of names, including bell curve, normal distribution, and Gaussian distribution. We’ll use the name **Gaussian** (pronounced GOW-see-an) in this course. The Gaussian is named for K. F. Gauss, a 19th-century pioneer in math and physics.

We won’t go into a lot of probability theory here, but the most important qualitative thing to observe about the Gaussian distribution is that it’s peaked around a central value, called the **mean**. In the graph above, the value of the mean is represented by the Greek letter $\mu$ (mu). The significance of this shape is that if you repeat a measurement lots of times, most of the values will be “close” to some central value, the mean.

We can be a little more quantitative about what we mean by “close”. A Gaussian distribution is characterized not only by its mean $\mu$, but by a second parameter called the **standard deviation**, represented by the Greek letter $\sigma$ (sigma). The standard deviation (which is always positive) is a measure of the width, or spread, of the distribution. The smaller $\sigma$ is, the more sharply peaked the distribution. A large value of $\sigma$ corresponds to a wide, flat distribution, making it harder to identify the peak.

**Rule of 68 and 95**

Using the mean and standard deviation, we can state the following numerical rules about measurements that follow a Gaussian distribution:

1. **68% of the time**, the measurement falls between $\mu - \sigma$ and $\mu + \sigma$, i.e., within one **standard deviation of the mean**. In other words, it has about 2/3 chance of being within “one sigma” of the mean.

2. **95% of the time**, it falls between $\mu - 2\sigma$ and $\mu + 2\sigma$. It is quite rare for a Gaussian variable to have a value that is more than 2 sigma from the mean. This is illustrated below:
Often we will say that we can be 95% confident that a value lies within 2 standard deviations (2\(\sigma\)) of the mean. This is the 95% confidence interval of the measurement.

For instance, if the cholesterol test has a standard deviation of 5 points, and we measure a cholesterol level of 200, then we could report:

- Cholesterol level: 200 ± 5 (68% CI)
- Cholesterol level: 200 ± 10 (95% CI)

The first statement says that 68% of the time, the true cholesterol level should be between 195 and 205—i.e., within 1\(\sigma\) of the mean. The second statement says that 95% of the time, the true cholesterol level should be between 190 and 210—i.e., within 2\(\sigma\) of the mean. **Both statements are telling you essentially the same thing, as long as we assume that the uncertainty has a Gaussian distribution.** It is common in many fields to report 95% confidence intervals, but you should always check explicitly to be sure.

**Standard Deviation and Standard Error**

Let’s assume that the cholesterol test has a standard deviation of 5 points. Is there anything you could do to get a better measurement of your true cholesterol level? Yes! You could **repeat the test multiple times, and take the average of the repeated measurements.**

Why does this help? Well, if each measurement could be a little bit high or a little bit low, when you take the average, those little variations will tend to cancel out. So you can report a smaller confidence interval for the average of the repeated measurements.

The uncertainty of the average (mean) of \(N\) measurements is called the **standard error**, and is given by the simple formula:

\[
\text{Standard error (SE)} = \frac{\sigma}{\sqrt{N}}
\]

In the example above, where \(\sigma = 5\) points, if you **repeat the measurement 25 times, the uncertainty will be reduced by a factor of 5!** In this case, you could report:

- Mean cholesterol level \((N = 25)\): 200 ± 1 (68% CI)
- Mean cholesterol level \((N = 25)\): 200 ± 2 (95% CI)
Experiment 1: Lab Activity

- Follow along in this lab activity. Wherever you see a question highlighted in red, be sure to answer that question, or paste in some data, or a graph, or whatever is being asked. Your lab report will be incomplete if any of these questions remains unanswered.

- **Introduction:**

In this experiment you’ll be playing with something called a Gauss gun, which is a remarkably simple device consisting of a magnet and some steel balls on a track. You can see some examples of the Gauss gun on YouTube:


We can ignore the details of the magnetic interactions (which are complicated) and just view the Gauss gun as a system that takes an “in” ball (the one rolling in) and spits out an “out” ball (the one that goes shooting out). It seems from the videos that the “out” ball is going much faster than the “in” ball… is that true?

You may notice that the magnet recoils when the “out” ball shoots out. Does this recoil have any effect on the speed of the “out” ball? You’ll see what happens when you hold the magnet (to stop it from recoiling). And so our ultimate goal in this experiment is to answer (with uncertainty!) the question:

**Does the Gauss gun shoot farther if you hold the magnet so it doesn’t recoil?**

In other words: when we hold the magnet, is it the same, or not the same? We’ll need to collect some statistics on this system in order to give a complete answer.

**Who are you? Take a picture of your lab group with Photo Both and paste it below along with your names.**

- **Materials:**

A magnet:

Some steel balls:

A long track with a ramp:

You’ll also have a long sheet of paper, some carbon paper, a 2-meter stick, and a sensor for the Logger Pro software that you can use to measure distances.
Set up the experiment

The Gauss gun will be set up to launch balls off the lab bench so they land on a sheet of white paper on the floor. Tape the paper to the floor so that the balls will land on it. Place a piece of carbon paper (dark side down) on the paper, and drop a ball on it. The ball should leave a mark at the spot where it hits the paper.

Once everything is aligned, clamp the ramp onto the table using the C-clamp (see right-hand image)

Now try rolling the ball from the ramp so it lands on the paper (just the ball; no Gauss gun yet). Roll it from different heights and see where it marks the paper.

What do you notice about the marks on the paper?

Test the uncertainty of the speed of the “in” ball

Now hold the ball right up against the plastic clip at the end of the ramp and let it roll onto the paper. Repeat this at least 20 times to see how much variation there is in where the ball marks the paper (you’ll want to have one person release the balls and another ready to catch them!)

What do you notice about the marks on the paper?

Build the Gauss gun

Line up the magnet with two balls right at the end of the track as shown. Then release a third ball from up against the plastic clip.

What happens?
• **Test the range of the gun with recoil**

Use the red carbon paper and mark the distance that the Gauss gun fires when you allow it to recoil. Repeat this 15-20 times to get a good distribution. Try to set up the experiment identically each time.

**What do you notice about the distribution of marks?**

• **Test the range of the gun without recoil**

Now use the blue carbon paper and mark the distance that the Gauss gun fires when one of you is holding the magnet so it won’t recoil. Repeat this 15-20 times to get a good distribution. Be sure that your finger is holding the magnet as shown so that you don’t touch the incoming or outgoing balls at all.

**What do you notice about the distribution of marks?**

• **Set up the equipment to measure the various distances**

Take the 2-meter stick and line it up along all the marks—that from the Gauss gun and from rolling the ball directly off the ramp. They should all lie roughly in a straight line. Draw a line along the meter stick.

Slide the meter stick so it is parallel to the line that you drew but be sure that it isn’t covering up any of the dots. Then tape the meter stick in place.

Place the Logger Pro motion sensor right against the stick near the end closest to the lab table. The sonar detector should be flipped “up” as shown below. Make sure that the switch inside is set on the “cart” setting, not on the “person/ball” setting.
Gauss Gun

• **Set up Logger Pro**

Open Logger Pro. Go to the File menu and open the Tutorials folder. As a group, work through the tutorials “01 Getting Started” and “07 Viewing Graphs.”

Open the “Experiment1.cmbl” file in the lab folder on the desktop. Logger Pro will start up and you’ll see the file for this experiment.

You’ll want to set up the sensor to collect distances when you tell it:

- Click on “Data Collection”
- Under “Mode” choose “Selected Events” and click OK.
- Now click the “Start Collection” button.

The sonar detector will start clicking and the display will show the distance from the detector to whatever object is in its range. Try moving your hand or foot around in front of the detector to see how it works.

• **Start collecting the “recoil allowed” data (red marks)**

Slide the plastic box right up against the edge of the meter stick. Make sure that nothing between the sonar sensor and the box. As you slide the box back and forth, the sensor should read the distance to the box.

Line up the box so its edge is just in line with one of the red dots, then click the “keep current value” icon on Logger Pro. The computer will record the distance to that dot. Repeat this for each of the red dots in the collection. Then stop the data collection (red button).

Once you have collected all of the data for the “recoil allowed” experiment, click on the graph in Logger Pro. Click the “statistics” icon and a box will show the mean and standard deviation for the collection of red dots.

**Record the mean and standard deviation here:**

• **Mark the mean and standard deviation**

Start collecting data again—be sure to click “store latest run.” This time be sure not to click the “keep current value” icon. Slide the plastic box until the Logger Pro distance is equal to the mean that you recorded above. Draw a line along the edge of the box and mark it as the mean. Then do the same thing to draw a line at the mean plus one standard deviation, and the mean minus one standard deviation. Then stop collecting.
How many of the points lie on or within $1\sigma$ of the mean? How many points should lie within this interval? Is there a discrepancy here? What does it mean?

• **Create a histogram of your data**

To make a histogram of your data, do the following:

- Highlight the column of the data you would like to include in the histogram
- Go to the menu bar on Logger Pro and click on “Insert”
- Scroll down to “Additional Graphs” and then select “Histogram”. A new graph with the histogram should appear.
- Double-click on the x-axis label of the histogram and make sure that the correct data set (the red “position” data) appears on the x-axis.
- If you add new data on the data column, the histogram should auto-update.
- You can adjust the number and size of the bins in the histogram by double-clicking on the x-bin axis of the histogram.
  - A new window will open.
  - On the bottom-left, you can set “Bin Size”, which determines the range of values to be included in each bin
  - On the bottom-left, you can set “Bin Start”, which determines the value for the initial bin

Then you’ll want to paste your histogram in this lab writeup. The easiest way to do this is with a “screen capture.” On the keyboard, press **command-shift-4**. You’ll see a little crosshairs that you can use to select the histogram on the screen. When you release the mouse button, the screen shot will appear on the desktop. Alternatively, use **command-control-shift-4** to save the image to the clipboard. Then paste the image into your lab write-up with **command-v**.

**Drag the screen shot of your histogram below:**

• **Calculate the 95% confidence interval of the mean**

Using the information provided in the background to this experiment, calculate the standard error of the mean, and use that to find the 95% confidence interval of the mean of this distribution. Mark that interval on the paper. You should end up with something that looks like what we have at right: a line for the mean, lines for $\pm 1\sigma$, and an interval showing the 95% CI of the mean.

**Calculation of the 95% confidence interval (show your work):**
Gauss Gun

• Repeat all of the calculations for the “no recoil” data (the blue data)

In Logger Pro, click “Start Collection” and do all of the same data collection and analysis for the “no recoil” data (the blue data): collect the data, find the mean and standard deviation, draw those on your paper, make a histogram, and show the 95% confidence interval for the mean of your distribution.

Mean and standard deviation (“no recoil”):

How many of the points lie on or within $1\sigma$ of the mean? How many points should lie within this interval? Is there a discrepancy here? What does it mean?

Histogram (“no recoil”):

Calculation of the 95% confidence interval (“no recoil”) (show your work):

Now tear out the portion of your paper that shows both the red and blue data. Hold the paper up to the computer’s camera and take a picture with Photo Booth. (You may need to select “Flip Photo” from the Edit menu if the photo is backwards.)

Paste the photo of your data, with the confidence intervals marked clearly, below:

• Conclusion

The big question! Can you be confident that the Gauss gun shoots farther when you prevent it from recoiling? Are you at least 95% confident? Why or why not?

On average, how much farther does it shoot?

What is the most important thing you learned in lab today?
Gauss Gun

What aspect of the lab was the most confusing to you today?